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Pulse Photomultiplier Simulation Based on the pn-i-pn Structure with Avalanche p-n Junction Barriers*

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ABSTRACT: A pulse amplification is modeled in a *pn-i-pn* structure-based photomultiplier having an internal avalanche-cascade amplification. The gain factor, the fast-action and the noise of the avalanche multiplication have been computed. The photomultipliers under investigation have proven to have a high gain factor, a low threshold of current sensitivity and a better operational reliability as compared with the avalanche photodiodes.

The theoretical and practical research on the alternative design of photomultipliers is being actuated by the growing demand for compact devices of an improved reliability, a high internal gain factor and a low noise level. Such photomultipliers are required for registering and measuring procedures in a variety of optical data processing systems, low-level radiation detection, range finding, navigation, and so on [1-5]. Nowadays, the most widely applied photodetectors with the internal amplification are the avalanche photodiodes (APD). They operate on the prebreakdown mode and are characterized by a high sensitivity, a high gain and fast-action. However, their application is restrained due to their high operational voltage and necessary stabilization [3]. The further improvement of APD characteristics is quite a challenge from the engineering point of view, since by increasing the gain factor one can neither keep the avalanche multiplication noise level low enough, nor provide a high repeatability and spatial homogeneity of the characteristics [1,3]. The reason is the sharp dependence between the coefficient of the electron and hole collision

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ionization and the electric field strength, as well as the growing probability of local microplasma formations by high voltages that results in the device failure, and the positive back-coupling between the avalanche processes in the multiplication layer of the $p-n$ junction due to the electrons and holes [3,6]. The probability of the local microplasma formations can be suppressed by employing the perfect-structured crystals [6], or due to the special design features, e.g. introducing of a local negative feedback between the avalanche initiating current and the gain factor [4,5]. Still the simplest way to reduce the avalanche multiplication noise, as well as to decrease the microplasma probability and to lower the requirements to the power supply stability is to apply a low bias at the $p-n$ junction. This method is inapplicable for APD, as the gain factor decreases with the lowering bias at the $p-n$ junction. On the contrary, it is just right for the $pn-i-pn$ structure-based photomultipliers with the reverse biased $p-n$ junctions, where the gain factor is determined not only by the bias at the $p-n$ junctions, but also by the number of the multiplication cascades in them [7]. Note that for the first time in papers [8,9] the reverse biased $pn-i-pn$ structures with the positive feedback have been recommended as the basis for designing high energy particle detectors and random oscillation sources.

The aim of the present work is the simulation of the internal amplification processes in the $pn-i-pn$ structure-based photomultipliers with the avalanche $p-n$ junctions, and the computation of the general characteristics of the device, i.e., gain factor, fast-action and noise.

PROBLEM STATEMENT

Figure 1 presents the diagram of the photomultiplier with the internal amplification and window in the photoresponsive p_1-n_1 junction deplection zone.

The optical power $P_{opt}(\omega)=P_{opt}\left(1+me^{j\omega t}\right)$ (ω is the modulation frequency, m is the modulation coefficient) is incident on the p_1-n_1 junction, while some portion $R_{opt}P_{opt}$ of the radiation can be reflected (R_{opt} is the reflection coefficient).

The advantage of such a location of the window is that the generation of the electron-hole pairs and their dissolution take place in the one and the same semiconductor volume. There are no losses for the pair recombination by their drift toward the $p-n$ junction, and no time is taken by this drift either, quite the reverse to the case when the window lays beyond the $p-n$ junction [3].

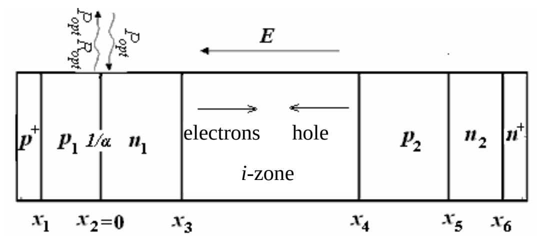

FIGURE 1. Diagram of the *pn-i-pn* structure-based photomultiplier with a reverse biased *p-n* junctions.

In the photosensitive cell of the $p-n$ junction barrier, the electron-hole pairs are built providing that the light energy exceeds that of the forbidden band. These values are related as λ (nm)=1240/*E* (eV). For a complete absorption, the depth of the photosensitive cell should be smaller than that of the radiation penetration $1/\alpha$ (α is the absorption coefficient) [1]. In germanium, starting from short waves through to $\lambda = 1.5$ µm, the radiation is absorbed virtually in full at the depth of 1÷2 µm from the surface [3]. In silicon, the spectral sensitivity peak falls at λ =0.9 µm, which corresponds to the radiation penetration depth of 30 µm [2,3]. The spectral photosensitivity region of gallium arsenide lays within the range 0.3÷0.9 µm with the spectral sensitivity peak at λ =0.9 µm. This radiation corresponds to the penetration depth of 1÷2 µm [1].

Let us consider the internal amplification process for a primary photocurrent in the $pn-i-pn$ structure with the positive back-coupling through the drift current between the avalanche $p-n$ junctions [7]. In the first p_1-n_1 junction the electron-hole pairs were generated by the radiation absorption $P_{opt}(\omega)$. Their number is determined by the quantum efficiency. These pairs are multiplied due to the collision ionization in the multiplication layer of the p_1-n_1 junction. In order for the collision ionization to take place, the thickness of the barrier region of the $p-n$ junction should exceed the free path length of the nonequilibrium carriers, and the energy accumulated by them in the junction region should exceed the threshold of the collision ionization of the lattice atoms. The electron-hole pairs built due to the collision ionization get dissolved under the influence of the electric field. The holes go to the contact p^+, while inducing current in the internal circuit. The electrons drift through the $i-$region toward the p_2-n_2 junction, where they initiate the collision ionization followed by a new turn of the electron-hole pairing. These pairs are dissolved again. The electrons go to the contact n^+, where they induce current in the internal circuit. The holes, through the $i-$region, return to the p_1-n_1 junction, where they initiate the collision ionization anew, and the electrons and holes are combined in pairs again, and so on. Hence, the pulse internal amplification is conditioned by the avalanche-cascade multiplication of the electron-hole pairs in the $p_1 - n_1$ and $p_2 - n_2$ junctions of the structure [7], and the induced current in the internal circuit of the avalanche-cascade photomultiplier (ACPM) increases stepwise in the course of time. The number of steps is determined by the number of the multiplication cascades in the $p - n$ junctions. It is seen from Fig. 1 that the collision ionization in the $p_1 - n_1$ junction is initiated by holes and in the $p_2 - n_2$ junction by electrons.

As a mathematical model for ACPM we have chosen the one-dimensional diffusion-drift model (DDM) that offers an adequate description to the collision ionization in the $p - n$ junctions [1,4]. The DDM equations in the dimensionless form are written as [6,7]:

$$\left.\begin{aligned} \frac{\partial E}{\partial x} &= \frac{q}{\varepsilon\varepsilon_0}(p - n + N), \\ \frac{\partial \varphi(x,t)}{\partial x} &= -E(x,t) \end{aligned}\right\}; \tag{1}$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + \alpha_n J_n + \alpha_p J_p - R(n,p); \tag{2}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + \alpha_n J_n + \alpha_p J_p - R(n,p); \tag{3}$$

$$J_n = qnv_n; \; J_p = qpv_p; \; \mathrm{J_{cm}} = \varepsilon\varepsilon_0 \frac{\partial E}{\partial t}; \tag{4}$$

$$J = J_n + J_p + J_d, \tag{5}$$

where E is the electric field strength, φ is the electric potential, J is the total current density, J_n is the electron current density, J_p is the hole current density, J_d is the bias current density, n is the electron concentration, p is the hole concentration, v_n, v_p is the velocity of the electrons and holes, respectively, $N(x) = \begin{cases} -N_{a1}, \; -L_{p1} < x < x_2; \; N_{d1}, \; x_2 < x < L_{n1} \\ -N_{a2}, \; L_{p2} < x < x_5; \; N_{d2}, \; x_5 < x < L_{n2}; \end{cases}$ is the foreign atom concentration, N_a is the acceptor concentration, N_d is the donor concentration, α_n, α_p are collision ionization coefficients of electrons and holes that are approximated by the exponential dependence on the field $\alpha(E) = A\exp[-(b/E)^m]$ [10]; the values of parameters *A*, *b* and *m* depend on the semiconductor material, $R(n, p)$ is the electron and hole recombination speed by the Shockley-Read-Hall formula [1]; L_p, L_n are dimensions of the barrier regions of the $p-n$ junctions.

The differential equations (1)-(3) are complemented by the due boundary conditions, the continuity conditions and the initial conditions:

$$\left.\begin{array}{l} E(-L_{p1},t)=0,\ \ E(L_{n1},t)=E_i(L_{n1,}t), \\ E(L_{p2},t)=E_i(L_{p2},t),\ \ E(L_{n2},t)=0 \end{array}\right\}; \tag{6}$$

$$\left.\begin{array}{l} \varphi(-L_{p1},t)=V,\ \ \varphi(L_{n1},t)=V_i+V_2, \\ \varphi(L_{p2},t)=V_2,\ \ \varphi(L_{n2},t)=0 \end{array}\right\}; \tag{7}$$

$$\left.\begin{array}{l} J_p(-L_{p1},t)=J(t)-J_{ns}(-L_{p1},t), \\ J_n(x_2,t)=I_0/S, \\ J_n(L_{n1,}t)=J(t)-J_{pi}(L_{n1},t), \\ J_p(L_{p2},t)=J(t)-J_{ni}(L_{p2},t), \\ J_n(L_{n2},t)=J(t)-J_{ps}(L_{n2},t) \end{array}\right\}; \tag{8}$$

$$\left.\begin{array}{l} E(x_{2,5}-0,t)=E(x_{2,5}+0,t), \\ \varphi(x_{2,5}-0,t)=\varphi(x_{2,5}+0,t) \end{array}\right\}; \tag{9}$$

$$J_{pi}(L_{n1},0)=J_{ps};\ \ J_{ni}(L_{p2},0)=J_{ns}, \tag{10}$$

where J_{pi} and J_{ni} are the densities of the electron and the hole currents, arriving from the *i*-region at the $p-n$ junction, I_0 is the current conditioned by the primary photocurrent, the background and the dark currents, S is the square of the p_1-n_1 junction, J_{ns} is the density of the electron dark current, J_{ps} is the density of the hole dark current.

GAIN FACTOR

For the numerical solution, the initial equations (1)-(10) have been normalized by the formulas: $\bar{E} = E / E_0$; $\bar{\varphi} = \varphi / \varphi_0$; $\bar{J} = J / J_0$; $\bar{n} = n / n_i$; $\bar{p} = p / n_i$; $\bar{N} = N / n_i$; $\bar{t} = t / t_0$; $\bar{x} = x / L_0$. The normalization values are $E_0 = \varphi_0 / L_0$, V/m; $L_0 = \sqrt{\varepsilon\varepsilon_0 \varphi_0 / q n_i}$, m; $J_0 = \dfrac{q n_i D_0}{L_0}$, A/M^2; $D_0 = 1$, m^2/s; $t_0 = L_0^2 / D_0$, s;

where n_i is the equilibrium electron concentration in the eigen semiconductor, T is the absolute temperature, q is the absolute value of the electron charge, $\varepsilon\varepsilon_0$ is the dielectric permittivity of the semiconductor, ε_0 is the free space dielectric permittivity, k is the Boltzmann constant (the overline above the dimensionless values is omitted).

The dimensionless DDM equations complemented by boundary conditions (6)-(8), continuity conditions (9) and initial conditions (10), are solved by applying the difference methods [11-16].

The gain factor of ACPM is determined by the expression [1, 6]

$$M = \prod_{j=1}^{K} m_{1j} m_{2j} \; ; \tag{11}$$

$$m_{1j} = \left\{ 1 - \int_{-L_{p1}}^{L_{n1}} \alpha_p \exp\left[- \int_{-L_{p1}}^{x} (\alpha_n - \alpha_p) dx' \right] dx \right\}_j^{-1} ; \tag{12}$$

$$m_{2j} = \left\{ 1 - \int_{L_{p2}}^{L_{n2}} \alpha_n \exp\left[- \int_{x}^{L_{p2}} (\alpha_n - \alpha_p) dx' \right] dx \right\}_j^{-1} , \tag{13}$$

where m_{1j}, m_{2j} are the coefficients of the $j-$th multiplication cascade in the $p_1 - n_1$ and $p_2 - n_2$ junctions, respectively, K is the number of the multiplication cascades for the time t. Formulas (11)-(13) allow for the fact that in the $p_1 - n_1$ junction the collision ionization is initiated by the holes, and in the $p_2 - n_2$ junction by the electrons (Fig. 1). The magnitude of the gain factor, according to (11), depends on the multiplication cascade number K and the gain factors of the $p - n$ junctions. It follows from formulas (11)-(13) that, by the denominator tending to unity, the gain factor increases unrestrictedly, and the avalanche current is limited by the internal circuit resistance only [1,6].

$$\int_{L_p}^{L_n} \alpha_n \exp\left[-\int_x^{L_p} (\alpha_n - \alpha_p)dx'\right]dx \to 1 \qquad (14)$$

Figure 2 shows the numerical results on the gain factor for ACPM made of various materials ($K = 8$). The ACPM gain factor in the avalanche mode evidently has a finite value. This limitation is determined by the effect of the mobile carrier charges upon the electric field. The dynamic amplification range of ACPM made of Ge, Si, GaAs reaches 80 dB and falls within a narrow voltage interval U/U_{av}. The position of this interval at the X-axis is determined by the doping level and the material properties, i.e., different kind of dependence between the collision ionization coefficients of Ge, Si and GaAs and the electric field [10].

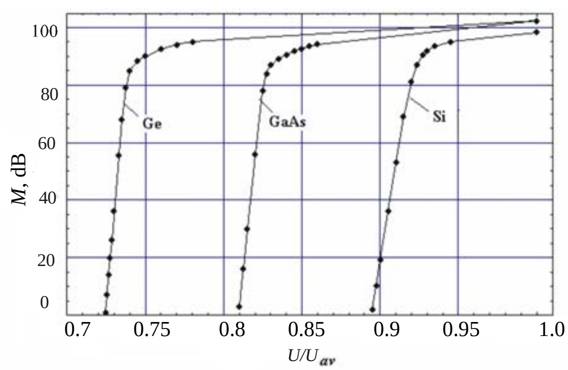

FIGURE 2. The internal amplification coefficient of Ge, Si and GaAs ACPM as a stress function allowing for the effect of the spatial charge on the electric field.

The gain factor in APD, for instance, whose back bias is the same as at the $p_2 - n_2$ junction in ACPM, is $m = M^{1/K} = 80^{1/8} \approx 1.7$ dB. Thus, the essential advantage of ACPM over APD is their high gain factor by a low bias. The use of low voltages enables one to decrease the probability of the local microplasma formation at the $p - n$ junction, to improve the reliability of ACPM and to reduce the requirements to the voltage stability as compared to APD.

FAST-ACTION OF THE AVALANCHE-CASCADE PHOTOMULTIPLIERS

The response time is determined by the run time of the collision ionization, the transit time the current carriers need to fly through the barrier regions of the $p-n$ junctions of the structure, the flight time of the electrons and holes through the $i-$region of the structure and the multiplication cascade number K. For a case when the carrier velocities in the barrier regions of the $p-n$ junctions and in the $i-$zone of the structure are constant, the response speed is determined by the expression

$$\tau = KT \,, \tag{15}$$

where $T = \tau_{j1n} + \tau_{dn} + \tau_{j2n} + \tau_{j2p} + \tau_{dp} + \tau_{j1p}$ is the structure period, τ_{j1n} is the electron drift time in the $p_1 - n_1$ junction, τ_{dn} is the electron drift time in the i - region, τ_{j2n} is the electron drift time in the $p_2 - n_2$ junction, τ_{j2p} is the hole drift time in the $p_2 - n_2$ junction, τ_{dp} is the hole drift time in the *i*-region, τ_{j1p} is the hole drift time in the $p_1 - n_1$ junction. Figure 3 illustrates the variations in the fast-action of a $pn-i-pn$-based Si-ACPM (15). As seen from Fig. 3, the response improves from 31.8 ns ($U_i = -0.25$ V) to 9 ns ($U_i = -0.75$ V) as the electric field in the *i*-region $E_i = U_i / d_i$ grows. The ACPM response speed is K times lower than that of APD. The response speed of ACPM can be increased by reducing the number of the interaction cascades and speeding up the current carrier drift in the *i*-region of the $pn-i-pn$ structure.

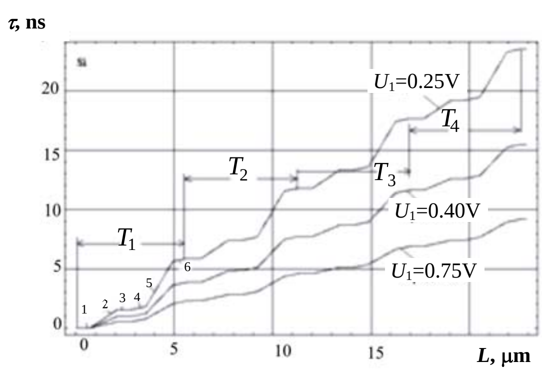

FIGURE 3. The drift time (response) τ of a $pn-i-pn$-based Si-ACPM as a function of the drift distance *L*, passed by the carriers.

As it follows from Fig. 3, the fast-action τ is determined by the period T and the multiplication cascade number K. Period T is determined by the time intervals $t_1 = \tau_{j1n}$; $t_2 = \tau_{dn}$; $t_3 = \tau_{j2n}$; $t_4 = \tau_{j2p}$; $t_5 = \tau_{dp}$ and $t_6 = \tau_{j1p}$, where the intervals t_2 and t_5 exceed by far the others. The reason is that in the barrier region of the both $p-n$ junctions the charge carriers move in a strong electric field, while their speed catches up with the saturation rate. At the same time, in the *i*-region the current carriers move in a weak electric field, so that their speed is less than the saturation rate. Besides, the dimension of the *i*-zone exceeds substantially that of the $p-n$ junction depletion zones.

In order for the pulse not to be overlapped during amplification, its duration τ_{imp} should be less than the half-period of the structure $\tau_{imp} < T/2$, and the pulse repetition cycle should exceed the response time τ. In the silicon ACPM in question the amplified pulses had the duration $\tau_{\rm imp} < 1 \div 2.5$ ns and the repetition rate $f_{imp} < 31 \div 100$ MHz. Note that a pulse, being amplified as a result of the collision ionization, stretches, and direct current is initiated [8,9]. This stretching can be eliminated by introducing catchers into the *i*-region, that narrow the pulse while recombining the electron-hole pairs.

SIGNAL-TO-NOISE RATIO

The root-mean-square (rms) power of an optical signal arriving through the window in the depletion zone of the avalanche $p_1 - n_1$ junction (Fig. 1) at a 100% modulation, is $P_{opt}/\sqrt{2}$. The rms value of the photocurrent after the avalanche amplification is determined by the expression [1]

$$i_p = q(\eta / h\nu)(P_{opt}/\sqrt{2})M, \qquad (16)$$

where $h\nu = 1.237q/\lambda(\mu\text{m})$ is the photon energy, η is the quantum efficiency that is the ratio of the number of the photogenerated electron-hole pairs to the number of incident photon.

The rms noise value of the avalanche-cascade amplification is determined as a sum of the rms noise values of every avalanche multiplication cascade in the $p-n$ junctions $\left\langle i_s^2 \right\rangle = \sum_{k=1}^{K} \left\langle i_s^2 \right\rangle_k$, $k = 1, 2, 3, ..., K$ [1]. In the case when the gain factors of all the multiplication cascades are $m_k = m$ and $\alpha_n/\alpha_p = \alpha_p/\alpha_n = k$, the avalanche noise of ACPM is determined by the expression [see Annex 1]

$$\left\langle i_s^2 \right\rangle = 2qI_0 M^2 F(M) B, \qquad (17)$$

where $F(M) = \frac{M-1}{m-1}(m/M)^2 F(m)$ is the noise factor of ACPM, and $F(m)$ is the noise factor of APD. Therefore, the excess noise of ACPM is $(M-1)/(m-1)(m/M)^2$ times lower than in APD. When $m = M$ holds, the noise factors of ACPM and APD coincide.

The termal noise released at an equivalent ohmic resistance R_{eq}, is described as [1]

$$\left\langle i_T^2 \right\rangle = 4kT\left(1/R_{eq}\right)B, \qquad (18)$$

where k is the Boltzmann constant, and T is the absolute temperature.

The signal-to-noise ratio is found from expressions (16)-(18) [1]

$$S/N = \frac{1/2\left(q\eta P_{opt}/h\nu\right)^2}{2qI_0 F(M)B + 4kTB/R_{eq}M^2}. \qquad (19)$$

The minimum optical power P_{opt} required for the prescribed ratio S/N to be true, is determined by

$$P_{opt} = \frac{2h\nu}{\eta}\frac{S}{N}F(M)B\left\{1+\left[1+\frac{I_{eq}}{qBF(M)^2\frac{S}{N}}\right]\right\}^{\frac{1}{2}}, \qquad (20)$$

where $I_{eq} \equiv \left(I_B + I_D\right)F\left(M\right) + 2kT/qR_{eq}M^2$.

A relative performance measure for photomultipliers is the noise equivalent power (*NEP*). It is defined as a rms power of the incident radiation that ensures the signal-to-noise ratio of unity within the frequency band of 1 Hz. From (20) we find [1]

$$NEP = 2\frac{h\nu}{\eta}F\left(M\right)\left[1+\left(1+\frac{I_{eq}}{qF\left(M\right)^2}\right)^{1/2}\right]. \qquad (21)$$

As follows from (21), the response of ACPM can be improved by increasing R_{eq} and decreasing m, I_B and I_D. Figure 4 shows the *NEP* behavior of a silicon ACPM and APD allowing for the restrictions imposed by the heat noise, the background and the dark current (the collision ionization ratio is $k = \alpha_{n,p} / \alpha_{p,n} = 0.1$, *K*=8).

It is evident from Fig. 4 that, by an equal overall gain factor, *NEP* of ACPM (solid lines 1-5) are more than by an order lower than APD (dotted lines 1-5). Thus, ACPM are low-noise devices with the threshold of current sensitivity more than by an order lower than that of APD.

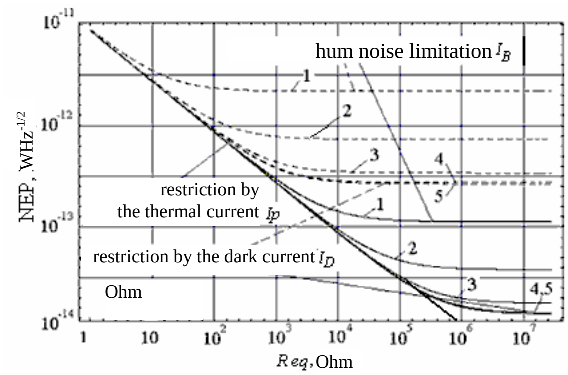

FIGURE 4. Dependence of *NEP* of Si ACPM (solid lines) and Si APD (dotted lines) v.v. the load resistance (the gain factor is *M*=141, the dark current is $I_D = 1.5*10^{-10}$ A, the background current is I_B: line 1- $I_B = 10^{-8}$; 2 - $I_B = 10^{-9}$; 3- $I_B = 10^{-10}$; 4- $I_B = 10^{-11}$, 5- $I_B = 10^{-12}$, A) .

CONCLUSIONS

Thus, the dynamic amplification range of ACPM reaches 80 dB. In the avalanche breakdown mode the gain is limited by the charge of mobile carriers. The fast-action of ACPM is several dozens of nanoseconds and is determined by the multiplication cascade number and the carrier drift time in the $pn - i - pn$ structure.

As compared with APD, the response speed shown by ACPM is *K*-times lower. An ACPM is a low-noise device; its *NEP* is more than by an order lower than that of APD. The purpose of ACPM is the amplification of the photocurrent pulses whose duration does not exceed the half-period of the structure, and their repetition cycle is longer then the fast-action.

ACPM is an innovative pulse photomultiplier of a rich potential, with the avalanche-cascade multiplication of the primary photocurrent taken as the operating principle.

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ANNEX. NOISE OF THE AVALANCHE-CASCADE AMPLIFICATION

The primary current $I_0 \equiv I_p + I_B + I_D$ arrives at a multiplication layer of the avalanche $p_1 - n_1$ junction, where $I_P = (q\eta / h\nu) P_{opt}$ is the primary photocurrent induced by an optical signal, $I_B [0,1]$ is a current induced by the background radiation, I_D is the dark current occurring due to the heat generation of the electron-hole pairs in the junction depletion zpne. In the $p_1 - n_1$ junction, a collision ionization takes place that is initiated by current I_0, which causes the multiplication of I_0 so that at the $p_1 - n_1$ junction output the current value is $I_1 = m_1 I_0$ (m_1 is the multiplication factor of the $p_1 - n_1$ junction, whose value is determined from the solution of DDM equations). The current arriving at the input of the $p_2 - n_2$ junction is I_1, at the output the current is $I_2 = m_2 I_1 = m_1 m_2 I_0$ (m_2 is the multiplication factor of the $p_2 - n_2$ junction, whose value is determined from the solution of DDM equations). The current I_2 arrives at the input of the $p_1 - n_1$ junction, at the output the current is $I_3 = m_3 I_2 = m_1 m_2 m_3 I_0$ (m_3 is the multiplication factor of the $p_1 - n_1$ junction). The avalanche current K of the cascade multiplication is $I_k = I_0 \prod_{k=1}^{K} m_k$. The noise generated by the first cascade is [1,2] $\langle i_s^2 \rangle_1 = 2qI_0 \langle m_1^2 \rangle B$, where B is the frequency band. The noise generated by the k-th cascade is $\langle i_s^2 \rangle_k = 2qI_{k-1} \langle m_k^2 \rangle B$. According to [17], the noise of a photoelectron multiplier having K multiplication cascades is $\langle i_s^2 \rangle = \sum_{k=1}^{K} \langle i_s^2 \rangle_k$. Correspondingly, the ACPM noise is

$$\langle i_s^2 \rangle = 2qI_0 \langle m_1^2 \rangle B + 2qI_1 \langle m_2^2 \rangle B + \cdots + 2qI_{K-1} \langle m_K^2 \rangle B$$

Taking $M = m^K$ out of the brackets, and assuming the multiplication factors $m_k = m_{k+1} = m$ of the both $p-n$ junctions to be the same, obtain

$$\left\langle i_s^2 \right\rangle = 2qI_0 \left\langle m^2 \right\rangle B\left(\frac{1}{m}+\frac{1}{m^2}+\cdots+\frac{1}{m^K}\right)M \; .$$

On summing the series in the brackets, we find the expression for the ACPM noise in the form

$$\left\langle i_s^2 \right\rangle = 2qI_0 \frac{M-1}{m-1} m^2 F(m) B \; ,$$

where $F(m) = \left\langle m^2 \right\rangle / m^2 = km + (2-1/m)(1-k)$ is the noise factor of the first multiplication cascade that is equal to the relation between the rms value of the multiplication factor m and its squared mean value, B is the frequency band. In the $p_1 - n_1$ junction, the collision ionization is initiated by the holes, in the $p_2 - n_2$ junction by the electrons (Fig. 1). Accordingly, $k = \alpha_\mathrm{p} / \alpha_\mathrm{n}$ is true for the $p_1 - n_1$ junction and $k = \alpha_n / \alpha_p$ - for the $p_2 - n_2$ junction. For the sake of simplicity we assume that $\alpha_n / \alpha_p = \alpha_p / \alpha_n = k$. Writing down the ACPM noise in the form $\left\langle i_s^2 \right\rangle = 2qI_0 M^2 F(M) B$, we obtain the expression for the ACPM noise factor in the form $F(M) = \dfrac{M-1}{m-1}\left(m/M\right)^2 F\left(m\right)$.