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Dielectric Response of Saturated Semiconductors[2])

By

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The optical properties of a direct-gap semiconductor are investigated under conditions of a strong optical pumping by an intense laser source of frequency slightly larger than the energy band gap. By taking into account the Coulomb interaction between electrons and holes we found that the energy gap in the quasi-particle spectrum diminishes. The dielectric constant of the system is calculated in the generalized Hartree-Fock approximation. The Raman cross-section is found to vanish for energies below 2Δ, where Δ is the quasi-particle energy gap.

Es werden die optischen Eigenschaften eines direkten Halbleiters untersucht under den Bedingungen eines starken optischen Pumpens mit einer intensiven Laserquelle, deren Frequenz wenig größer als die Energiebandlücke ist. Unter Berücksichtigung der Coulombwechselwirkung zwischen Elektronen und Löchern wird gefunden, daß die Energielücke im Quasiteilchenspektrum verschwindet. Die dielektrische Konstante des Systems wird in der verallgemeinerten Hartree-Fock-Näherung berechnet. Es wird gefunden, daß der Raman-Wirkungsquerschnitt für Energien unterhalb 2Δ verschwindet, wobei Δ die Quasiteilchenenergiebandlücke ist.

1. Introduction

Studies of semiconductor behaviour under strong optical pumping are presently the object of particular attention. Considerable interest has been attached to the question of the possible occurrence of a Bose condensate of bound electron–hole pairs when in conditions of strong photoinjection. Such ideas were advanced by Blatt and Böer [1a] and by Moskalenko [1b]. In the case of indirect-gap semiconductors the situation seems to have been settled, with Keldysh [2] prediction of formation of metallic droplets of electrons and holes being confirmed through considerable accumulation of experimental evidence, as well as theoretical studies [3]. There seems to exist some evidence pointing to the formation of a Bose-like condensation of the electron–hole plasma (EHP) in direct-gap semiconductors as well [4].

Another kind of Bose-type condensation may occur in saturated semiconductors, i.e. when an electromagnetic wave with frequency slightly larger than the band gap is strong enough to produce equalization of the rates of photon absorption and recombination. In the resulting steady state the quasi-Fermi levels for electrons and holes are clamped together with a separation equal to the exciting laser energy [5]. Galitskii et al. [6] have shown that under saturation conditions an excitonic-like phase of the EHP can be stabilized. The new phase consists of a bound electron–hole pair of momentum $\boldsymbol{q}$, the wave vector of the incident photon. The binding energy is proportional to the laser intensity and, therefore, it is a phase transition induced by the optical pump. The saturation condition can be achieved if the binding energy is larger than the collision frequency of the carriers, and the inverse of the recombination time. In a previous communication [7] we derived the grand-partition function of the saturated semiconductor (in the absence of Coulomb interactions), and showed that the EHP condensates in a non-zero momentum excitonic Bose phase.

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Once the possibility of the condensation is established, one needs to determine the observable effects of the new phase. Optical measurements can be a useful tool for that purpose, and the dielectric constant of the system provides the relevant information. For that reason we proceed to evaluate the dielectric constant of the saturated semiconductor, including Coulomb interaction between carriers in the time-dependent Hartree-Fock approximation. The same approximation is used to calculate, using the equation of motion method, the excitation spectrum of the quasi-particles in the Bose condensate. It is shown that the inclusion of the electron–hole attractive Coulomb interaction tends to reduce the binding energy of the radiation induced exciton-like quasi-particles.

In Section 2 we present the general theory and the calculation of the renormalized quasi-particle spectrum, in Section 3 we derive the RPA dielectric constant of the saturated semiconductor. In Section 4 the results are summarized.

2. General Theory

We consider a two-band semiconductor in the electromagnetic field of vector potential

$$\boldsymbol{A} = \boldsymbol{A}_0 \cos(\omega t - \boldsymbol{q}\cdot\boldsymbol{r})\,; \qquad \boldsymbol{A}\cdot\boldsymbol{q} = 0 \tag{1}$$

whose Hamiltonian, in the effective mass approximation, is given by

$$H = H_0 + H_c\,, \tag{2}$$

where

$$H_0 = \sum_{\boldsymbol{p}} \{E_e(\boldsymbol{p})\, c^+_{\boldsymbol{p}} c_{\boldsymbol{p}} + E_h(\boldsymbol{p})\, h_{-\boldsymbol{p}} h^+_{-\boldsymbol{p}} + \lambda_{\boldsymbol{pq}} c^+_{\boldsymbol{p}} h^+_{-\boldsymbol{p}-\boldsymbol{q}}\, e^{-i\omega t} + \lambda^*_{\boldsymbol{pq}} h_{-\boldsymbol{p}-\boldsymbol{q}} c_{\boldsymbol{p}}\, e^{i\omega t}\} \tag{3}$$

and

$$H_c = \frac{1}{2} \sum_{\boldsymbol{pp'k}} V(\boldsymbol{k})\, \{c^+_{\boldsymbol{p}} c^+_{\boldsymbol{p'}} c_{\boldsymbol{p'}+\boldsymbol{k}} c_{\boldsymbol{p}-\boldsymbol{k}} + h^+_{-\boldsymbol{p}} h^+_{-\boldsymbol{p'}} h_{-\boldsymbol{p'}-\boldsymbol{k}} h_{-\boldsymbol{p}+\boldsymbol{k}} + 2 c^+_{\boldsymbol{p}} h^+_{-\boldsymbol{p'}} h_{-\boldsymbol{p'}-\boldsymbol{k}} c_{\boldsymbol{p}-\boldsymbol{k}}\}\,. \tag{4}$$

Here $E_e(\boldsymbol{p})$ and $E_h(\boldsymbol{p})$ are the energy of electrons and holes measured from the centre of the band gap; $c^+_{\boldsymbol{p}}(h^+_{-\boldsymbol{p}})$ and $c_{\boldsymbol{p}}(h_{-\boldsymbol{p}})$ are the creation and annihilation operators for electrons (holes), respectively, $\lambda_{\boldsymbol{p},\boldsymbol{q}}$, is the electric dipole matrix element for the interband transition, and $V(\boldsymbol{q})$ are the matrix element of the Coulomb interaction in the plane-wave approximation for the wave functions. Regarding the electrons and holes as independent particles we have omitted the transitions of the electrons from one band to the other. We have also neglected the electric dipole matrix element for intraband transition, since they are small far from the plasma frequency [8] (this is true when a centre of symmetry exists in the crystal).

We consider the case $\lambda_{\boldsymbol{p},\boldsymbol{q}}$ real, for simplicity, and we take the case $\boldsymbol{q} = \boldsymbol{0}$; this limit has been worked out in [7]. We perform a unitary transformation $U(t)$, to make the Hamiltonian (2) time-independent:

$$U(t) = \exp\left\{-\frac{1}{2}\, i\omega t \sum_{\boldsymbol{k}} (c^+_{\boldsymbol{k}} c_{\boldsymbol{k}} + h^+_{-\boldsymbol{k}} h_{-\boldsymbol{k}})\right\}. \tag{5}$$

This transformation introduces the chemical potential $\mu = \mu_e + \mu_h = \hbar\omega$, which fixes the average photocarrier concentration in the saturated state, and where μ_e and μ_h are the quasi-Fermi levels of electrons and holes, respectively. Only H_0 is altered by this transformation, and it becomes

$$H_0 = U^+(t)\, H_0 U(t) - i U^+(t)\, \frac{\partial U(t)}{\partial t}\,, \tag{6}$$

$$H_0 = \sum_{\boldsymbol{p}} \{\varepsilon_e(p)\, c^+_{\boldsymbol{p}} c_{\boldsymbol{p}} + \varepsilon_h(\boldsymbol{p})\, h^+_{-\boldsymbol{p}} h_{-\boldsymbol{p}} + \lambda_{\boldsymbol{p}} (c^+_{\boldsymbol{p}} h^+_{-\boldsymbol{p}} + h_{-\boldsymbol{p}} c_{\boldsymbol{p}})\}\,, \tag{6a}$$

where $\varepsilon_e(\boldsymbol{p})$ and $\varepsilon_h(\boldsymbol{p})$ are given by

$$\varepsilon_{e(h)}(\boldsymbol{p}) = \frac{p^2}{2m_{e(h)}} + \frac{1}{2}(E_g - \omega), \tag{7}$$

where E_g is the energy band gap.

We calculate the equation of motion of the electron and hole operators, including the Coulomb interactions in the Hartree-Fock approximation. The new spectrum of quasi-particles found is formally equal to the non-perturbed case, that is

$$\omega_{\boldsymbol{p}}^{\pm} = \tfrac{1}{2}[\widetilde{E}_e(\boldsymbol{p}) - \widetilde{E}_h(\boldsymbol{p})] \pm \{\tfrac{1}{4}[\widetilde{E}_e(\boldsymbol{p}) + \widetilde{E}_h(\boldsymbol{p})]^2 + \Delta_{\boldsymbol{p}}^2\}^{1/2}, \tag{8}$$

where

$$\widetilde{E}_{e,h}(\boldsymbol{p}) = \varepsilon_{e,h}(\boldsymbol{p}) - \sum_{\boldsymbol{k}} V(\boldsymbol{k} - \boldsymbol{p})\, n_{\boldsymbol{k}}^{e,h}, \tag{9}$$

$$\Delta_{\boldsymbol{p}} = \lambda_{\boldsymbol{p}} - \sum_{\boldsymbol{k}} V(\boldsymbol{k} - \boldsymbol{p})\, b_{\boldsymbol{k}}, \tag{10}$$

$$n_{\boldsymbol{k}}^{e} = \langle c_{\boldsymbol{k}}^{+} c_{\boldsymbol{k}} \rangle, \tag{11}$$

$$n_{\boldsymbol{k}}^{h} = \langle h_{-\boldsymbol{k}}^{+} h_{-\boldsymbol{k}} \rangle, \tag{12}$$

and

$$b_{\boldsymbol{k}} = \langle c_{\boldsymbol{k}}^{+} h_{-\boldsymbol{k}}^{+} \rangle = \langle h_{-\boldsymbol{k}} c_{\boldsymbol{k}} \rangle. \tag{13}$$

Notice that the new energy gap, $\Delta_{\boldsymbol{p}}$, in the quasi-particle spectrum decreases when Coulomb interaction is taken into account, i.e. it tends to decrease the stability of the induced excitonic phase against the usual free-exciton gas phase.

3. The Dielectric Constant

To calculate the dielectric constant we will follow the procedure of Nozières and Pines [9]. We introduce in the Hamiltonian of the system a weak longitudinal oscillating electric field in the form of an oscillating test charge of wave vector $\boldsymbol{Q}$ and frequency Ω, whose charge density is assumed to be

$$e\{r_{\boldsymbol{Q}}\, e^{-i(\Omega t - \boldsymbol{Q}\cdot\boldsymbol{r})} + \text{c.c.}\}. \tag{14}$$

Therefore, the interaction Hamiltonian is

$$H' = V(\boldsymbol{Q})\{\varrho(\boldsymbol{Q})\, r_{\boldsymbol{Q}}\, e^{-i\Omega t} + \text{c.c.}\}\, e^{-\eta|t|}, \tag{15}$$

where η is introduced to insure the adiabatic application of the test charge. Here $\varrho(\boldsymbol{Q})$ is the charge density operator defined as

$$\varrho(\boldsymbol{Q}) = -e \sum_{\boldsymbol{k}} (c_{\boldsymbol{k}+\boldsymbol{Q}}^{+} c_{\boldsymbol{k}} + h_{-\boldsymbol{k}}^{+} h_{-\boldsymbol{k}-\boldsymbol{Q}}) = \varrho_e(\boldsymbol{Q}) + \varrho_h(\boldsymbol{Q}). \tag{16}$$

Following [9] we define the dielectric constant as

$$\varepsilon(\boldsymbol{Q}, \Omega) - 1 = \frac{-\varrho(\boldsymbol{Q})}{\varrho(\boldsymbol{Q}) + r_{\boldsymbol{Q}}\, e^{-i\Omega t}}. \tag{17}$$

In order to calculate the function $\varrho(\boldsymbol{Q})$ we define the following operators

$$\varrho_{\boldsymbol{k},e}^{\boldsymbol{Q}} = c_{\boldsymbol{k}+\boldsymbol{Q}}^{+} c_{\boldsymbol{k}}, \tag{18a}$$

$$\varrho_{\boldsymbol{k},h}^{\boldsymbol{Q}} = h_{-\boldsymbol{k}}^{+} h_{-\boldsymbol{k}-\boldsymbol{Q}}, \tag{18b}$$

$$b_{\boldsymbol{k}}^{\boldsymbol{Q}} = h_{-\boldsymbol{k}-\boldsymbol{Q}} c_{\boldsymbol{k}}, \tag{18c}$$

$$b_{\boldsymbol{k}}^{-\boldsymbol{Q}} = c_{\boldsymbol{k}+\boldsymbol{Q}}^{+} h_{-\boldsymbol{k}}^{+}. \tag{18d}$$

We calculate the equation of motion of these operators in the generalized RPA approximation [10], which turn out to be

$$\begin{aligned}[H, \varrho_{\boldsymbol{k},\mathrm{e}}^{\boldsymbol{Q}}] = {} & (\widetilde{E}_{\boldsymbol{k}+\boldsymbol{Q}}^{\mathrm{e}} - \widetilde{E}_{\boldsymbol{k}}^{\mathrm{e}})\, \varrho_{\boldsymbol{k},\mathrm{e}}^{\boldsymbol{Q}} + \Delta_{\boldsymbol{k}+\boldsymbol{Q}} b_{\boldsymbol{k}}^{\boldsymbol{Q}} - \Delta_{\boldsymbol{k}} \bar{b}_{\boldsymbol{k}}^{\boldsymbol{Q}} - \\ & - (n_{\boldsymbol{k}+\boldsymbol{Q}}^{\mathrm{e}} - n_{\boldsymbol{k}}^{\mathrm{e}})\, V(\boldsymbol{Q})\, [\varrho_{\mathrm{e}}(\boldsymbol{Q}) + \varrho_{\mathrm{h}}(\boldsymbol{Q})] + \\ & + b_{\boldsymbol{k}} \sum_{\boldsymbol{q}} V(\boldsymbol{q} - \boldsymbol{k})\, \bar{b}_{\boldsymbol{q}}^{\boldsymbol{Q}} - b_{\boldsymbol{k}+\boldsymbol{Q}} \sum_{\boldsymbol{q}} V(\boldsymbol{q} - \boldsymbol{k})\, b_{\boldsymbol{q}}^{\boldsymbol{Q}}\,, \end{aligned} \tag{19a}$$

$$\begin{aligned}[H, \varrho_{\boldsymbol{k},\mathrm{h}}^{\boldsymbol{Q}}] = {} & (\widetilde{E}_{\boldsymbol{k}}^{\mathrm{h}} - \widetilde{E}_{\boldsymbol{k}+\boldsymbol{Q}}^{\mathrm{h}})\, \varrho_{\boldsymbol{k},\mathrm{h}}^{\boldsymbol{Q}} + \Delta_{\boldsymbol{k}} b_{\boldsymbol{k}}^{\boldsymbol{Q}} - \Delta_{\boldsymbol{k}+\boldsymbol{Q}} \bar{b}_{\boldsymbol{k}}^{\boldsymbol{Q}} - \\ & - (n_{\boldsymbol{k}}^{\mathrm{h}} - n_{\boldsymbol{k}+\boldsymbol{Q}}^{\mathrm{h}})\, V(\boldsymbol{Q})\, [\varrho_{\mathrm{e}}(\boldsymbol{Q}) + \varrho_{\mathrm{h}}(\boldsymbol{Q})] + \\ & + b_{\boldsymbol{k}+\boldsymbol{Q}} \sum_{\boldsymbol{q}} V(\boldsymbol{q} - \boldsymbol{k})\, \bar{b}_{\boldsymbol{q}}^{\boldsymbol{Q}} - b_{\boldsymbol{k}} \sum_{\boldsymbol{q}} V(\boldsymbol{q} - \boldsymbol{k})\, b_{\boldsymbol{q}}^{\boldsymbol{Q}}\,, \end{aligned} \tag{19b}$$

$$\begin{aligned}[H, b_{\boldsymbol{k}}^{\boldsymbol{Q}}] = {} & - (\widetilde{E}_{\boldsymbol{k}}^{\mathrm{e}} + \widetilde{E}_{\boldsymbol{k}+\boldsymbol{Q}}^{\mathrm{h}})\, b_{\boldsymbol{k}}^{\boldsymbol{Q}} + \Delta_{\boldsymbol{k}+\boldsymbol{Q}}\, \varrho_{\boldsymbol{k},\mathrm{e}}^{\boldsymbol{Q}} + \Delta_{\boldsymbol{k}} \varrho_{\boldsymbol{k},\mathrm{h}}^{\boldsymbol{Q}} - \\ & - (b_{\boldsymbol{k}+\boldsymbol{Q}} + b_{\boldsymbol{k}})\, V(\boldsymbol{Q})\, [\varrho_{\mathrm{e}}(\boldsymbol{Q}) + \varrho_{\mathrm{h}}(\boldsymbol{Q})] - \\ & - (1 - n_{\boldsymbol{k}+\boldsymbol{Q}}^{\mathrm{h}} - n_{\boldsymbol{k}}^{\mathrm{e}}) \sum_{\boldsymbol{q}} V(\boldsymbol{q} - \boldsymbol{k})\, b_{\boldsymbol{q}}^{\boldsymbol{Q}}\,, \end{aligned} \tag{19c}$$

$$\begin{aligned}[H, \bar{b}_{\boldsymbol{k}}^{\boldsymbol{Q}}] = {} & (\widetilde{E}_{\boldsymbol{k}+\boldsymbol{Q}}^{\mathrm{e}} + \widetilde{E}_{\boldsymbol{k}}^{\mathrm{h}})\, \bar{b}_{\boldsymbol{k}}^{\boldsymbol{Q}} - \Delta_{\boldsymbol{k}} \varrho_{\boldsymbol{k},\mathrm{e}}^{\boldsymbol{Q}} - \Delta_{\boldsymbol{k}+\boldsymbol{Q}} \varrho_{\boldsymbol{k},\mathrm{h}}^{\boldsymbol{Q}} + \\ & + (b_{\boldsymbol{k}+\boldsymbol{Q}} + b_{\boldsymbol{k}})\, V(\boldsymbol{Q})\, [\varrho_{\mathrm{e}}(\boldsymbol{Q}) + \varrho_{\mathrm{h}}(\boldsymbol{Q})] + \\ & + (1 - n_{\boldsymbol{k}}^{\mathrm{h}} - n_{\boldsymbol{k}+\boldsymbol{Q}}^{\mathrm{e}}) \sum_{\boldsymbol{q}} V(\boldsymbol{q} - \boldsymbol{k})\, \bar{b}_{\boldsymbol{q}}^{\boldsymbol{Q}}\,, \end{aligned} \tag{19d}$$

where

$$\varrho_{\mathrm{e(h)}}(\boldsymbol{Q}) = \sum_{\boldsymbol{k}} \varrho_{\boldsymbol{k},\mathrm{e(h)}}^{\boldsymbol{Q}}\,. \tag{20}$$

As we are working in the saturated state, it is easier to work in the set of operators defined by Bogolyubov [11], since there are no quasi-particles in the ground state. Hence, we perform a canonical transformation

$$\left.\begin{aligned} c_{\boldsymbol{k}} &= u_{\boldsymbol{k}} \alpha_{\boldsymbol{k}} + v_{\boldsymbol{k}} \beta_{\boldsymbol{k}}^{+}\,, \\ h_{-\boldsymbol{k}}^{+} &= - v_{\boldsymbol{k}} \alpha_{\boldsymbol{k}} + u_{\boldsymbol{k}} \beta_{\boldsymbol{k}}^{+} \end{aligned}\right\} \tag{21}$$

which, with the condition

$$u_{\boldsymbol{k}} v_{\boldsymbol{k}}\, (\widetilde{E}_{\boldsymbol{k}}^{\mathrm{e}} + \widetilde{E}_{\boldsymbol{k}}^{\mathrm{h}}) + \Delta_{\boldsymbol{k}} (u_{\boldsymbol{k}}^2 - v_{\boldsymbol{k}}^2) = 0\,, \tag{22}$$

diagonalizes the linearized Hamiltonian. Equation (22) corresponds to the minimization of the ground state energy in terms of the variational parameters u and v, subject to the normalization condition $u^2 + v^2 = 1$. We obtain the new set of equations of motion

$$[H, \alpha_{\boldsymbol{k}+\boldsymbol{Q}}^{+} \alpha_{\boldsymbol{k}}] = (\omega_{\boldsymbol{k}+\boldsymbol{Q}}^{-} - \omega_{\boldsymbol{k}}^{-})\, \alpha_{\boldsymbol{k}+\boldsymbol{Q}}^{+} \alpha_{\boldsymbol{k}}\,, \tag{23a}$$

$$[H, \beta_{\boldsymbol{k}}^{+} \beta_{\boldsymbol{k}+\boldsymbol{Q}}] = - (\omega_{\boldsymbol{k}+\boldsymbol{Q}}^{-} - \omega_{\boldsymbol{k}}^{-})\, \beta_{\boldsymbol{k}}^{+} \beta_{\boldsymbol{k}+\boldsymbol{Q}}\,, \tag{23b}$$

$$\begin{aligned}[H, \alpha_{\boldsymbol{k}+\boldsymbol{Q}}^{+} \beta_{\boldsymbol{k}}^{+}] = {} & (\omega_{\boldsymbol{k}+\boldsymbol{Q}}^{-} + \omega_{\boldsymbol{k}}^{+})\, \alpha_{\boldsymbol{k}+\boldsymbol{Q}}^{+} \beta_{\boldsymbol{k}}^{+} + m(\boldsymbol{k}, \boldsymbol{Q})\, \varrho(\boldsymbol{Q})\, V(\boldsymbol{Q}) + \\ & + \tfrac{1}{2}\, n(\boldsymbol{k}, \boldsymbol{Q})\, B_{\boldsymbol{k}}(\boldsymbol{Q}) - \tfrac{1}{2}\, l(\boldsymbol{k}, \boldsymbol{Q})\, A_{\boldsymbol{k}}(\boldsymbol{Q})\,, \end{aligned} \tag{23c}$$

$$\begin{aligned}[H, \beta_{\boldsymbol{k}+\boldsymbol{Q}} \alpha_{\boldsymbol{k}}] = {} & - (\omega_{\boldsymbol{k}+\boldsymbol{Q}}^{+} + \omega_{\boldsymbol{k}}^{-})\, \beta_{\boldsymbol{k}+\boldsymbol{Q}} \alpha_{\boldsymbol{k}} - m(\boldsymbol{k}, \boldsymbol{Q})\, \varrho(\boldsymbol{Q})\, V(\boldsymbol{Q}) - \\ & - \tfrac{1}{2}\, n(\boldsymbol{k}, \boldsymbol{Q})\, B_{\boldsymbol{k}}(\boldsymbol{Q}) - \tfrac{1}{2}\, l(\boldsymbol{k}, \boldsymbol{Q})\, A_{\boldsymbol{k}}(\boldsymbol{Q})\,, \end{aligned} \tag{23d}$$

where $\omega_{\boldsymbol{k}}^{\pm}$ are given in (8) and can alternatively be written

$$\omega_{\boldsymbol{k}}^{+} = \widetilde{E}_{\boldsymbol{k}}^{\mathrm{h}} u_{\boldsymbol{k}}^2 - \widetilde{E}_{\boldsymbol{k}}^{\mathrm{e}} v_{\boldsymbol{k}}^2 - 2 \Delta_{\boldsymbol{k}} u_{\boldsymbol{k}} v_{\boldsymbol{k}}\,, \tag{24}$$

$$\omega_{\boldsymbol{k}}^{-} = \widetilde{E}_{\boldsymbol{k}}^{\mathrm{e}} u_{\boldsymbol{k}}^2 - \widetilde{E}_{\boldsymbol{k}}^{\mathrm{h}} v_{\boldsymbol{k}}^2 - 2 \Delta_{\boldsymbol{k}} u_{\boldsymbol{k}} v_{\boldsymbol{k}}\,, \tag{25}$$

the new operators

$$A_{\boldsymbol{k}}(\boldsymbol{Q}) = \sum_{\boldsymbol{p}} V(\boldsymbol{p} - \boldsymbol{k}) \, (b_{\boldsymbol{p}}^{\boldsymbol{Q}} - \bar{b}_{\boldsymbol{p}}^{\boldsymbol{Q}}) \,, \tag{26}$$

$$B_{\boldsymbol{k}}(\boldsymbol{Q}) = \sum_{\boldsymbol{p}} V(\boldsymbol{p} - \boldsymbol{k}) \, (b_{\boldsymbol{p}}^{\boldsymbol{Q}} + \bar{b}_{\boldsymbol{p}}^{\boldsymbol{Q}}) \,, \tag{27}$$

and the factors

$$m(\boldsymbol{k}, \boldsymbol{Q}) = u_{\boldsymbol{k}} v_{\boldsymbol{k}+\boldsymbol{Q}} + u_{\boldsymbol{k}} v_{\boldsymbol{k}+\boldsymbol{Q}} \,, \tag{28a}$$

$$l(\boldsymbol{k}, \boldsymbol{Q}) = u_{\boldsymbol{k}} u_{\boldsymbol{k}+\boldsymbol{Q}} + v_{\boldsymbol{k}} v_{\boldsymbol{k}+\boldsymbol{Q}} \,, \tag{28b}$$

$$n(\boldsymbol{k}, \boldsymbol{Q}) = u_{\boldsymbol{k}} u_{\boldsymbol{k}+\boldsymbol{Q}} - v_{\boldsymbol{k}} v_{\boldsymbol{k}+\boldsymbol{Q}} \,. \tag{28c}$$

Introducing the interaction Hamiltonian given by (15) the equations of motion (23c) and (23d) remain the same if we replace $\varrho(\boldsymbol{Q})$ by $(\varrho(\boldsymbol{Q}) + r_{\boldsymbol{Q}} \, \mathrm{e}^{-i\Omega t + \eta|t|} + \text{c.c.})$. Assuming that a fluctuation on the test charge causes a fluctuation on the charge density $\varrho(\boldsymbol{Q})$ of the system, we obtain equations for the collective variables $\varrho(\boldsymbol{Q})$, $A_{\boldsymbol{k}}(\boldsymbol{Q})$, and $B_{\boldsymbol{k}}(\boldsymbol{Q})$:

$$\varrho(\boldsymbol{Q}) = \sum_{\boldsymbol{k}} [m(\boldsymbol{k}, \boldsymbol{Q}) \, S_{\boldsymbol{k}}^{-}(\boldsymbol{Q}, \Omega) \, \{ V(\boldsymbol{Q}) \, [\varrho(\boldsymbol{Q}) + r_{\boldsymbol{Q}} \, \mathrm{e}^{-i\Omega t + \eta|t|}] \, m(\boldsymbol{k}, \boldsymbol{Q}) + \\ + \tfrac{1}{2} n(\boldsymbol{k}, \boldsymbol{Q}) \, B_{\boldsymbol{k}}(\boldsymbol{Q}) \} - \tfrac{1}{2} l(\boldsymbol{k}, \boldsymbol{Q}) \, A_{\boldsymbol{k}}(\boldsymbol{Q}) \, m(\boldsymbol{k}, \boldsymbol{Q}) \, S_{\boldsymbol{k}}^{+}(\boldsymbol{Q}, \Omega)] \,, \tag{29}$$

$$A_{\boldsymbol{k}}(\boldsymbol{Q}) = - \sum_{\boldsymbol{p}} l(\boldsymbol{p}, \boldsymbol{Q}) \, V(\boldsymbol{k} - \boldsymbol{p}) \, [\{ V(\boldsymbol{Q}) \, [\varrho(\boldsymbol{Q}) + r_{\boldsymbol{Q}} \, \mathrm{e}^{-i\Omega t + \eta|t|}] \, m(\boldsymbol{p}, \boldsymbol{Q}) + \\ + \tfrac{1}{2} n(\boldsymbol{p}, \boldsymbol{Q}) \, B_{\boldsymbol{p}}(\boldsymbol{Q}) \} \, S_{\boldsymbol{p}}^{+}(\boldsymbol{Q}, \Omega) - \tfrac{1}{2} l(\boldsymbol{p}, \boldsymbol{Q}) \, A_{\boldsymbol{p}}(\boldsymbol{Q}) \, S_{\boldsymbol{p}}^{-}(\boldsymbol{Q}, \Omega)] \,, \tag{30}$$

$$B_{\boldsymbol{k}}(\boldsymbol{Q}) = \sum_{\boldsymbol{p}} n(\boldsymbol{p}, \boldsymbol{Q}) \, V(\boldsymbol{k} - \boldsymbol{p}) \, [\{ V(\boldsymbol{Q}) \, [\varrho(\boldsymbol{Q}) + r_{\boldsymbol{Q}} \, \mathrm{e}^{-i\Omega t + \eta|t|}] \, m(\boldsymbol{p}, \boldsymbol{Q}) + \\ + \tfrac{1}{2} n(\boldsymbol{p}, \boldsymbol{Q}) \, B_{\boldsymbol{p}}(\boldsymbol{Q}) \} \, S_{\boldsymbol{p}}^{-}(\boldsymbol{Q}, \Omega) - \tfrac{1}{2} l(\boldsymbol{p}, \boldsymbol{Q}) \, A_{\boldsymbol{p}}(\boldsymbol{Q}) \, S_{\boldsymbol{p}}^{+}(\boldsymbol{Q}, \Omega)] \,, \tag{31}$$

where

$$S_{\boldsymbol{k}}^{\pm}(\boldsymbol{Q}, \Omega) = [-\hbar\Omega - (\omega_{\boldsymbol{k}+\boldsymbol{Q}}^{-} + \omega_{\boldsymbol{k}}^{+}) - i\eta]^{-1} \pm [-\hbar\Omega + (\omega_{\boldsymbol{k}+\boldsymbol{Q}}^{+} + \omega_{\boldsymbol{k}}^{-}) - i\eta]^{-1} \,. \tag{32}$$

As ϱ, $A_{\boldsymbol{k}}$, and $B_{\boldsymbol{k}}$ are proportional to $r_{\boldsymbol{Q}} \, \mathrm{e}^{-i\Omega t}$ we need to determine the proportionality constants

$$A_{\boldsymbol{k}}(\boldsymbol{Q}) = \alpha_{\boldsymbol{k}}(\boldsymbol{Q}) \, V(\boldsymbol{Q}) \, [\varrho(\boldsymbol{Q}) + r_{\boldsymbol{Q}} \, \mathrm{e}^{-i\Omega t}] \,, \tag{33a}$$

$$B_{\boldsymbol{k}}(\boldsymbol{Q}) = \beta_{\boldsymbol{k}}(\boldsymbol{Q}) \, V(\boldsymbol{Q}) \, [\varrho(\boldsymbol{Q}) + r_{\boldsymbol{Q}} \, \mathrm{e}^{-i\Omega t}] \,. \tag{33b}$$

Substituting in the system of equations the matrix elements of the interaction between electrons and holes by a constant value U/N, i.e. assuming a statistically screened interaction of short-range order,

$$\langle V(\boldsymbol{p} - \boldsymbol{k}) \rangle \approx \frac{U}{N} \,, \tag{34}$$

the $\boldsymbol{k}$-dependence in $\alpha_{\boldsymbol{k}}(\boldsymbol{Q})$ and $\beta_{\boldsymbol{k}}(\boldsymbol{Q})$ can be dropped, and we can write for $\varrho(\boldsymbol{Q})$ the following expression:

$$\varrho(\boldsymbol{Q}) = [\varrho(\boldsymbol{Q}) + r_{\boldsymbol{Q}} \, \mathrm{e}^{-i\Omega t}] \, [F(\boldsymbol{Q}) + L(\boldsymbol{Q}) \, \beta(\boldsymbol{Q}) - D(\boldsymbol{Q}) \, \alpha(\boldsymbol{Q})] \,, \tag{35}$$

where

$$F(\boldsymbol{Q}) = \sum_{\boldsymbol{k}} m^2(\boldsymbol{k}, \boldsymbol{Q}) \, S_{\boldsymbol{k}}^{-}(\boldsymbol{Q}, \Omega) \, V(\boldsymbol{Q}) \,, \tag{36a}$$

$$L(\boldsymbol{Q}) = \sum_{\boldsymbol{k}} \tfrac{1}{2} n(\boldsymbol{k}, \boldsymbol{Q}) \, m(\boldsymbol{k}, \boldsymbol{Q}) \, S_{\boldsymbol{k}}^{-}(\boldsymbol{Q}, \Omega) \, V(\boldsymbol{Q}) \,, \tag{36b}$$

$$D(\boldsymbol{Q}) = \sum_{\boldsymbol{k}} \tfrac{1}{2} l(\boldsymbol{k}, \boldsymbol{Q}) \, m(\boldsymbol{k}, \boldsymbol{Q}) \, S_{\boldsymbol{k}}^{+}(\boldsymbol{Q}, \Omega) \, V(\boldsymbol{Q}) \,, \tag{36c}$$

$$\alpha(\boldsymbol{Q}) = \frac{f_l + R f_n}{1 + PR}, \tag{37a}$$

$$\beta(\boldsymbol{Q}) = \frac{f_n - P f_l}{1 + PR}. \tag{37b}$$

Here

$$f_n = U \sum_{\boldsymbol{k}} n(\boldsymbol{k}, \boldsymbol{Q})\, m(\boldsymbol{k}, \boldsymbol{Q}) \frac{S_{\boldsymbol{k}}^-(\boldsymbol{Q}, \Omega)}{1 - G(\boldsymbol{Q})}, \tag{38a}$$

$$f_l = U \sum_{\boldsymbol{k}} l(\boldsymbol{k}, \boldsymbol{Q})\, m(\boldsymbol{k}, \boldsymbol{Q}) \frac{S_{\boldsymbol{k}}^-(\boldsymbol{Q}, \Omega)}{1 - T(\boldsymbol{Q})}, \tag{38b}$$

$$P(\boldsymbol{Q}) = U \sum_{\boldsymbol{k}} \frac{1}{2} l(\boldsymbol{k}, \boldsymbol{Q})\, n(\boldsymbol{k}, \boldsymbol{Q}) \frac{S_{\boldsymbol{k}}^+(\boldsymbol{Q}, \Omega)}{1 - G(\boldsymbol{Q})}, \tag{38c}$$

$$R(\boldsymbol{Q}) = U \sum_{\boldsymbol{k}} \frac{1}{2} n(\boldsymbol{k}, \boldsymbol{Q})\, l(\boldsymbol{k}, \boldsymbol{Q}) \frac{S_{\boldsymbol{k}}^-(\boldsymbol{Q}, \Omega)}{1 - T(\boldsymbol{Q})}, \tag{38d}$$

$$G(\boldsymbol{Q}) = U \sum_{\boldsymbol{k}} \tfrac{1}{2} n^2(\boldsymbol{k}, \boldsymbol{Q})\, S_{\boldsymbol{k}}^-(\boldsymbol{Q}, \Omega), \tag{38e}$$

$$T(\boldsymbol{Q}) = U \sum_{\boldsymbol{k}} \tfrac{1}{2} l^2(\boldsymbol{k}, \boldsymbol{Q})\, S_{\boldsymbol{k}}^+(\boldsymbol{Q}, \Omega). \tag{38f}$$

Substituting in (33) we find for the dielectric constant

$$1 - \varepsilon(\boldsymbol{Q}) = F(\boldsymbol{Q}) + \frac{f_n[L(\boldsymbol{Q}) - D(\boldsymbol{Q})\, R(\boldsymbol{Q})] - f_l[D(\boldsymbol{Q}) + L(\boldsymbol{Q})\, P(\boldsymbol{Q})]}{1 + P(\boldsymbol{Q})\, R(\boldsymbol{Q})}. \tag{39}$$

To study the asymptotic behaviour of $\varepsilon(\boldsymbol{Q})$ for large Ω, we will make the approximation $m_e = m_h$. It is easy to see that S^+ behaves as Ω^{-1}, while S^- behaves as Ω^{-2}. $\alpha(\boldsymbol{Q})$ and $\beta(\boldsymbol{Q})$ both behave as $1/\Omega$, therefore, $1 - \varepsilon(\boldsymbol{Q})$ can be well approximated by $F(\boldsymbol{Q})$. Analyzing the imaginary part of the inverse of the dielectric constant, we have information on the behaviour of the scattering cross-section. It is easy to see that

$$\operatorname{Im} \varepsilon^{-1}(\boldsymbol{Q}, \Omega) = \frac{1}{2} V(\boldsymbol{Q}) \operatorname{Im} \sum_{\boldsymbol{k}} \left(1 - \frac{\xi_{\boldsymbol{k}}\xi_{\boldsymbol{k}+\boldsymbol{Q}} - \Delta_{\boldsymbol{k}}\Delta_{\boldsymbol{k}+\boldsymbol{Q}}}{\varepsilon_{\boldsymbol{k}}\varepsilon_{\boldsymbol{k}+\boldsymbol{Q}}}\right) S_{\boldsymbol{k}}^-(\boldsymbol{Q}, \Omega). \tag{40}$$

Using the relation

$$\lim_{\eta \to +0} (x \pm i\eta)^{-1} = pv \frac{1}{x} \mp i\pi\delta(x)$$

we find

$$\operatorname{Im} \varepsilon^{-1}(\boldsymbol{Q}, \Omega) = \frac{\pi}{2} V(\boldsymbol{Q}) \sum_{\boldsymbol{k}} \left(1 - \frac{\xi_{\boldsymbol{k}}\xi_{\boldsymbol{k}+\boldsymbol{Q}} - \Delta_{\boldsymbol{k}}\Delta_{\boldsymbol{k}+\boldsymbol{Q}}}{\varepsilon_{\boldsymbol{k}}\varepsilon_{\boldsymbol{k}+\boldsymbol{Q}}}\right) \delta(\hbar\Omega - (\omega_{\boldsymbol{k}+\boldsymbol{Q}}^+ + \omega_{\boldsymbol{k}}^-)) \tag{41}$$

(this can be done in all cases when $1/m_e > 1/m_h$, that is to throw away the term with $\delta(\hbar\Omega + \omega_{\boldsymbol{k}} + \omega_{\boldsymbol{k}+\boldsymbol{Q}})$. In order to estimate the shape of the Raman spectrum we will make the assumption $m_e = m_h$, then we obtain, after some rearrangement of variables,

$$\operatorname{Im} \varepsilon^{-1}(\boldsymbol{Q}, \Omega) = \frac{m^2}{2\pi^2 Q} \int\limits_{\Delta}^{\Omega - \Delta} \mathrm{d}E \, \frac{E(\Omega - E) + \Delta^2}{(E^2 - \Delta^2)^{1/2} \, [(\Omega - E)^2 - \Delta^2]^{1/2}}. \tag{42}$$

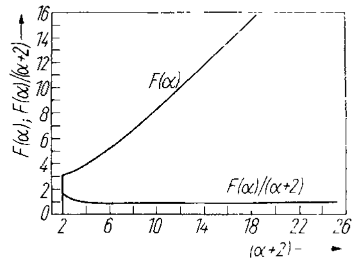

Fig. 1. Reduced Raman scattering cross-section, where $F(\alpha)$ is given by (44) and $F(\alpha)/(\alpha + 2)$ is the Raman scattering cross-section divided by the normal metal cross-section, here $\alpha = (\hbar\Omega - 2\Delta)/\Delta$

Performing this integration results

$$\operatorname{Im} \varepsilon^{-1}(\boldsymbol{Q}, \Omega) = \begin{cases} 0 & \text{for} \quad \Omega < 2\Delta \\ \dfrac{m^2}{2\pi^2 Q}\,\Delta F(\alpha) & \text{for} \quad \Omega > 2\Delta\,, \end{cases} \tag{43}$$

where

$$F(\alpha) = (\alpha + 4)\, E\left(\frac{\alpha}{\alpha + 4}\right) - 4\left(\frac{\alpha + 2}{\alpha + 4}\right) K\left(\frac{\alpha}{\alpha + 4}\right) \tag{44}$$

with $\alpha = (\Omega - 2\Delta)/\Delta$ and E and K are the elliptic functions. A graph of $F(\alpha)$ is shown in Fig. 1.

In the limit $\Delta \to 0$ our results tend to the limit of the normal metal, i.e.

$$\operatorname{Im} \varepsilon^{-1}(\boldsymbol{Q}, \Omega) = \frac{m^2 \Omega}{2\pi^2 Q}\,. \tag{45}$$

In conclusion, the scattering can occur only for frequencies above twice the quasi-particle energy gap. As we have neglected the Coulomb interaction between electrons or holes in the calculation of the dielectric constant, we obtained an equation similar to that of the scattering cross-section except for the lack of a term $|\varepsilon|^2$ in the denominator. This factor acts as a screening in the scattering cross-section of the single particle, and introduces a new line in the spectrum of the scattered light at a frequency ω_p, given by $\varepsilon(\boldsymbol{Q}, \omega_p) = 0$, due to the scattering by collective excitations.

4. Conclusions

We have investigated the behaviour of a direct-gap semiconducting system under high-intensity optical pumping. The strong electromagnetic field of the pumping beam must have a frequency larger than the band gap. We have found that adding Coulomb interaction to this system does not alter the formal dependence of the quasi-particle spectrum, however, the energy of the electrons and holes and the gap of the quasi-particle spectrum are significantly changed by inclusion of this interaction. The change in the energy gap shows that the stability of the induced excitonic phase diminishes. We have calculated the dielectric constant of the system in the RPA approximation and found that Raman scattering can occur only for frequencies above twice the energy gap of the quasi-particle spectrum.

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