Chaotic instability of currents in a reverse biased multilayered structure

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(Received 2 December 1996; accepted for publication 27 August 1997)

A new principle to generate chaotic signals using the phenomenon of charge avalanche multiplication and internal feedback in multilayered semiconductor structures is suggested. Linear and nonlinear theories for the self-oscillations are developed and existence of the chaotic regime with fast decay of correlations is proven.
[S0003-6951(97)00643-8]

The problem of generating chaotic oscillations has generated great interest in fields that range from ultrawide band radars[1] to secure communications.[2] Therefore, it becomes important to find semiconductor electronic devices, other than simple nonlinear circuits of low dimensionality, where relatively narrow band chaotic oscillations can be produced. Impact ionization in a **pn** junction is a well known phenomenon, used, for example, in IMPATT diodes to generate single frequency and chaotic oscillations.[3] Nonconventional ways of generating chaotic oscillations using impact ionization phenomenon in more complicated, multilayer semiconductor structures have been studied both theoretically and experimentally in a series of papers,[4] where chaotic motion of localized current structures (filaments) due to their nonliner diffusion in the transverse direction were treated. Here we consider a multilayered semiconductor structure, with *two areas of avalanche multiplication* of charge carriers due to impact ionization with the aim of the theoretical description of chaotic self-oscillations of the drift electron–hole current in asymmetrically doped reverse biased multilayered structure, where conditions for chaos can be provided.

The semiconductor structure under consideration consists of two **pn** junctions, connected through an intrinsic semiconductor and brought close to the avalanche breakdown threshold via reverse biased voltage (see Fig. 1). We consider the conductivity of the intrinsic region much larger than that of the depleted regions and at the same time, much less than the conductivity of the **p** and **n** layers. Therefore, the distribution of the electric field strength across this structure has two peaks around the physical **pn** junctions (Fig. 1). In such a structure the avalanche multiplication will take place only inside the depleated areas. The dynamics of the charge and current densities and the **E** field in classical semiconductor devices is governed by a self-consistent system of partial differential equations consisting of the full current and the continuity and Poisson's equations. We restrict ourselves to considering the simplest, still realistic, model which is able to describe the chaotic current instabilities we are interested in. For a detailed derivation of the model equations we refer the reader to Refs. 5 and 6; here we give a qualitative description of the device and write down the final equations. We assume that the semiconductor sample is uniformly doped perpendicularly to the flow of charges. This assumption allows us to consider it as a one-dimensional problem. Besides, under the condition of reverse bias the diffusion current can also be neglected compared to the drift current. The charge particle recombination term is neglected, since the product of the generated electron and hole densities is much smaller than the square of the intrinsic charge density. Finally, we assume that the drift velocities of the charge carriers do not depend on the coordinates within each of the layers because the electric field is assumed not to vary, which is a reasonable assumption if the **i** region is homogeneous. Under these conditions the continuity equations for each region of the structure can be written in a similar form:

$$\frac{\partial n(x,t)}{\partial t}+v_n\frac{\partial n(x,t)}{\partial x}=F(n,p),$$

$$\frac{\partial p(x,t)}{\partial t}-v_p\frac{\partial p(x,t)}{\partial x}=F(n,p), \qquad (1)$$

where $n(x,t)(p(x,t))$, $v_n(v_p)$, and $\alpha_n(E)(\alpha_p(E))$ denote the charge density, velocity, and ionization coefficients, respectively, for electrons (holes). The function $F(n,p)$ is different for the different regions of the structure. Since there is no impact ionization inside the intrinsic region and both relaxation and recombination of carriers are neglected, $F(n,p)=0$ for that case, while for the depletion slabs $F(n,p)=\alpha_n v_n n(x,t)+\alpha_p v_p p(x,t)$.

A self-consistent description of the spatiotemporal behavior of the drift current in the device under consideration can be separated into two independent problems. The first one describes a free propagation of the electrons and holes inside the **i** region with the alternative transformation of the charge densities at the boundaries, i.e., narrow depletion slabs. This corresponds to the solution of Eq. (1) having $F(n,p)=0$ with both initial and boundary conditions. The second describes the process of the charged particle avalanche multiplication inside the depletion slabs, which corresponds to the solution of Eq. (1) with given initial and

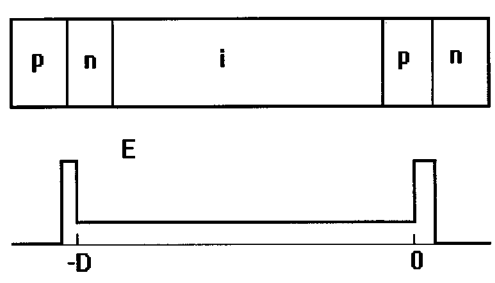

FIG. 1. Schematic view of a reverse biased multilayered structure cross section and internal E field distribution.

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boundary conditions for the incoming charge density. Solving this problem we find the dependence of the charge density coming out of the avalanche area on the incoming one and can use it as boundary conditions in the first problem.

In the left **pn** junction, for which the electron and hole velocities are equal, the space charge does not change while impact ionization takes place. Thus the space charge does not increase and, hence, there is no nonlinear dependence of the charge coming out of the slab on the incoming one within the working range of the current. According to the physical nature of the charge avalanche multiplication[7] we can write the following boundary condition at that slab

$$n^{(2)}(\varsigma=-1,\tau)=M_p p^{(2)}(\varsigma=-1,\tau), \tag{2}$$

where $\tau=t(v_n+v_p)/2d$ and $\varsigma=x/D$ are dimensionless time and spatial coordinates; D is the distance between the boundaries of the depletion slabs; M_p is the coefficient of avalanche multiplication of the holes inside the first slab, which does not depend on the hole density. Hereafter, the indices 1, 2, and 3 denote variables for the left slab, **i** region, and right slab, respectively. According to the results of Refs. 5 and 6, in order to provide the means for a chaotic instability of the drift current to exist, we need to find the conditions such that make the dependence of the charge density coming out of the avalanche area on the incoming one nonlinear logisticlike for one of the slabs. Let us assume that in the right **pn** junction the **p** layer is much wider than the **n** layer. In this case, the depletion slab will be concentrated inside the **p** region. Besides, let us consider a semiconductor for which the ionization coefficient (IC) and drift velocity for electrons are much larger than for holes. In this slab we should observe a decrease in the total electric field while the space charge is being increased due to both impact ionization and simultaneous fast sweeping electrons out of the depletion slab. Since there is a strong dependence of the IC on the electric field, there will be a decrease of the dependence of the current coming out of that slab on the incoming one. Therefore we are providing the required nonlinear dependence discussed previously.

To obtain the nonlinear boundary conditions at this slab we need, first, to solve Eq. (1) having $F(n,p)=\alpha_n v_n n(x,t)$, with both boundary

$$n^{(3)}(+0,\tau)=n^{(2)}(-0,\tau) \quad \text{and} \quad p^{(3)}(\delta,\tau)=n^{(3)}(\delta,\tau) \tag{3}$$

and initial

$$\begin{aligned} n^{(3)}(\varsigma,0)&=n^{(2)}(\tau_n\varsigma,0):\tau\in[0,\tau_n),\\ p^{(3)}(\varsigma,0)&=0:\tau\in[\tau_n,\tau_p) \end{aligned} \tag{4}$$

conditions and second, to take into account the electron IC dependence on the density of generated holes. In Eq. (4) $\tau_{n(p)}=(v_n+v_p)\delta/2v_{n(p)}$ is the dimensionless electron (hole) time of flight throughout the right slab. The solution for this initial-boundary-value problem shows exponential growth of the hole density with distance and depends on the incoming charges via convolution integrals. Using this solution, assuming that the majority of the charges is generated just at the end of the depletion slab at the right and putting $p^{(2)}\times(0,\tau)=p^3(0,\tau)$ we obtain the necessary boundary condition, connecting the hole density coming out of the second depletion slab with the incoming electron density:

$$p^{(3)}(0,\tau'+\tau_0)=M_n[n^{(3)}(0,\tau')]n^{(3)}(0,\tau'), \tag{5}$$

where $\tau'=\tau-\tau_0$; $M_n[n^{(3)}]=(1+\alpha_n(E)d)e^{\alpha_n(E)d}$ is the coefficient of avalanche multiplication of the electrons inside the right depletion slab, which depends nonlinearly on the hole density. For the most used semiconductors, Si and Ge, the dependence of IC on the electric field can be well approximated by $\alpha_n(E)=\exp[-E^0/E]$, where α_0 and E^0 are the constants (see for example Ref. 7).

The generated holes compensate partially the space charge in the depletion slab due to fast sweeping of electrons out of the slab. If E_0 is the electric field strength due to the bias voltage drop across the depletion slab the total averaged electric field determining the value of IC is $E=E_0-ap^{(3)}$, where the last term is the solution of the Poisson's Equation averaged over the p layer, for the homogeneously distributed hole density. The coefficient a is determined by the parameters of the **pn** junction such as geometry, doping and should be specified for a concrete device. Using these expressions we can write

$$M_n[n^{(3)}]=\alpha_0 d[1+\alpha_0 dF(\delta E,n^{(3)})]e^{\alpha_0 d\, F(\delta E,n^{(3)})}, \tag{6}$$

where $F(\delta E,n^{(3)})=\exp[-1/(1+\delta E-n^{(3)}A)]$ and $A=a/E^0$; $\delta E=E_0/E^0$. Note that in Eq. (6) we used the equality $n^{(3)}=p^{(3)}$, which is valid for the electron–hole pairs generation.

We are interested in the solution of the problem for large values of time. Therefore $\tau_0\ll\tau$ in Eq. (5) and we can expand Eq. (5) in series over the small parameter τ_0. As a result, we get the following approximate boundary condition at the second slab:

$$\tau_0\frac{dp^{(2)}(0,\tau)}{d\tau}+p^{(2)}(0,\tau)=M_n[n^{(2)}(0,\tau)]n^{(2)}(0,\tau). \tag{7}$$

Setting $\tau_0=0$, we find the nonlinear reflection coefficient of the electrons on the second slab without taking into account the avalanche rising time.

Since for hyperbolic equations the solution is constant on the characteristics, the initial-boundary problem, given by Eq. (1) with $F(n,p)=0$, boundary conditions (2) and (7), and initial conditions similar to Eq. (4), can be reduced to the following difference-differential equation:[5,6]

$$\begin{aligned}\tau_0\frac{dn^{(2)}(0,\tau)}{d\tau}&+n^{(2)}(0,\tau)\\ &=M_pM_n[n^{(2)}(0,\tau-\Theta)]n^{(2)}(0,\tau-\Theta),\end{aligned} \tag{8}$$

where $\Theta=\Theta_n+\Theta_p$, $\Theta_{n(p)}=\tau_{n(p)}v_{n(p)}/v^i_{n(p)}\delta$, and $v^i_{n(p)}$ is the electron (hole) drift velocity in the **i** region.

The solution of the linearized Eq. (8) defines the charge density behavior in the equilibrium state and is $n(0,\tau)=n(0,0)\exp(\gamma\tau)$, where γ is the root of its characteristic equation $\tau_0\gamma=-1+M_pM_n[0]\exp(-\gamma\Theta)$. The latter equation has no analytic solutions and in general it has to be solved numerically. However, when $\gamma\Theta\ll1$ we can estimate

$$\gamma\simeq\frac{\gamma_0}{1+\tau_0/[\exp(\gamma_0\Theta)-1]\gamma_0}, \tag{9}$$

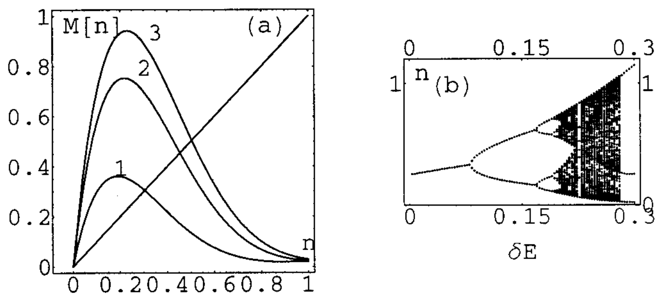

FIG. 2. (a) Nonlinear dependence of the avalanche multiplication of holes ($M[n]$) on the incoming electron density, the curves 1, 2, and 3 correspond to $\delta E = 0.05$, 0.2, and 0.3, respectively; (b) bifurcation diagram as a function of δE for the corresponding one-dimensional map: $\alpha_0 d = 10$.

where $\gamma_0 = \ln(M_p M_n[0])/\Theta$ is the solution of the characteristic equation for $\tau_0 = 0$.

We can see from Eq. (9) that the thermal fluctuations of the charge density in the structure will grow with time ($\gamma > 0$), if the product $M_p M_n[0]$ exceeds 1, in good agreement with the physics of the problem. The rate of growth of the charge density, determined by γ, has a stronger dependence on the distance between avalanche areas and on the charge mobilities in the **i** area than on the electron and hole avalanche multiplication coefficients.

If $\tau_0 \ll 1$, solutions for Eq. (8) will be close to the solution of the difference equation obtained from Eq. (8) by setting $\tau_0 = 0$ (Refs. 8 and 9). Solutions for that difference equation, in turn, are defined by the properties of the one-dimensional map, obtained from this equation by discretizing time. In our case this map is defined by Eq. (6), the normalized curves of which for different values of δE are shown in Fig. 2(a). The bifurcation diagram as a function of δE (which corresponds to varying the voltage applied across the structure) is shown in Fig. 2(b). It is seen that the period doubling route to chaos takes place as a function of the voltage applied. Therefore, we should expect a chaotic solution for Eq. (8) for $\delta E > 0.22$. The corresponding solutions have been obtained numerically as the limit of the evolution according to Eq. (8) of the initial distribution of the charge density (see Fig. 3). The Lyapunov exponent calculated for the solution of Eq. (8) is positive for $\delta E > 0.22$, while the cross-section of phase space diagram in the delayed coordinates is homogeneously filled for the chaotic oscillations. The inertiality of the avalanche reflection of the charges in the depletion slab leads to the narrowing of the Fourier spectrum of the generated chaotic signals, as well as to the smoothing of their autocorrelation function. However, for small avalanche delays ($\tau_0 < 0.01$) we can obtain chaotic signals with wide spectrum bandwidth and fast decay of correlations.[8,9] This is equivalent to making the ratio between the width of the avalanche area and that of the intrinsic region small, for the device under consideration.

Positive Lyapunov exponent, continuous Fourier spectrum, fast decay of correlations, and homogeneous filling of the phase space clearly show that real chaotic oscillations of current could be generated in the semiconductor device under study. In particular, the generated chaotic signals have

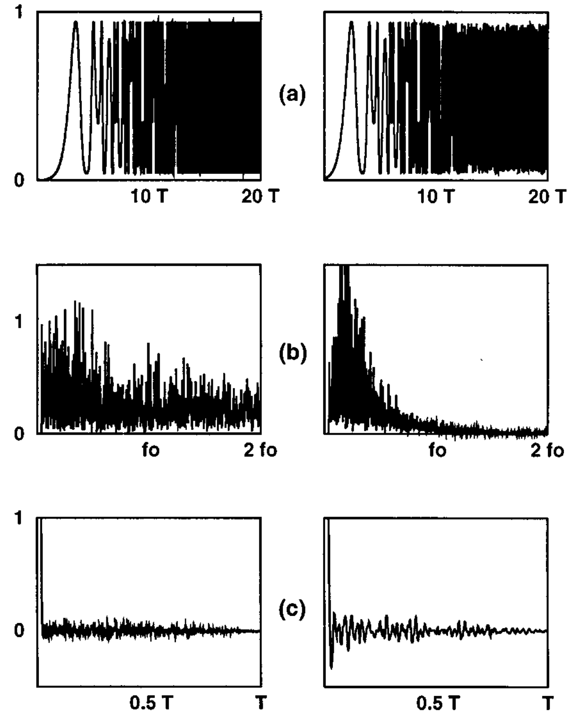

FIG. 3. Solution of Eq. (9) showing: (a) transient to chaotic oscillations (b) power spectrum density for the steady-state chaotic regime, and (c) autocorrelation function, for the case of fully developed chaos ($\delta E = 0.25$) for $\tau_0 = 0$ (left) and $\tau_0 = 0.002$ (right): $T = D/d$; $f_0 = 1/2T$.

very fast decay of correlations, which means that they could be used for many applications.

In conclusion, we have been able to show a new principle to generate wide band chaotic signals using the phenomenon of charge avalanche multiplication and internal feedback in semiconductor structures considered in this work. This principle could be used to develop a source of chaotic oscillations, which can be useful as a noise signal generator with wide frequency spectrum and fast decay of correlations, of great importance for fields such as Noise Radar, Secure Communications, Metrology, and others.

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