# Current Oscillations in Avalanche Particle Detectors with p-n-i-p-n-Structure 

Konstantin A. Lukin, Hilda A. Cerdeira, and Alberto A. Colavita


#### Abstract

We describe the model of an avalanche high energy particle detector consisting of two p-n-junctions, connected through an intrinsic semiconductor with a reverse biased voltage applied. We show that this detector is able to generate the oscillatory response on the single particle passage through the structure. The possibility of oscillations leading to chaotic behavior is pointed out.


## I. INTRODUCTION

THE sensitivity of a semiconductor particle detector, as well as that of a photodiode, is enhanced using avalanche multiplication of the charge carriers in depleted areas of p-n-junctions [1]-[3]. Efforts have been directed in finding ways to increase the multiplication rate and, at the same time, to decrease the avalanche excess noise [3]. The reason for the avalanche excess noise lies on the presence of the internal feedback due to hole-impact-ionizing generation of the electron-hole pairs. To suppress this generation many different methods have been proposed [3]-[8]: employment of the large difference between ionization energies for electrons and holes in graded-gap materials in [4]; use of the large enough difference of the effective impact-ionization rates for electrons and holes in multiquantum-well or staircase avalanche photodiodes (APD), [5], [6]; resonance enhancement of the impact ionization, induced by the zone folding effect, used for superlattice APD in [7]. A method to eliminate parasitic feedback by trapping holes in a potential well formed between two heterojunctions, in which the avalanche electron-hole pairs generation takes place was proposed in [8].

For particle detectors the more conventional approach is to use impact ionization in the wide enough depleted region of a p-i-n-diode structures [1], [9]. Cerdeira et al. considered the positive feedback caused by the holes impact ionization process, to organize the oscillatory response of a semiconductor avalanche particle detectors to a pulse-like external perturbation [10]. They proposed to use a semiconductor structure with two narrow depleted areas (slabs), separated by an intrinsic semiconductor with high conductivity. The avalanche multiplication will take place inside the depleted

[^0]areas. The electrons generated move from one slab to another through the intrinsic region, while the holes move in the inverse direction. The strength of the electric field inside this region was considered too small to generate electronhole pairs. When the electrons reach the second depleted slab, they will be accelerated by the local $\mathbf{E}$-field, causing a new avalanche multiplication. Meanwhile, the cloud of holes moves toward the first depleted slab inside the i-region without any change and will induce the next avalanche multiplication inside the first depleted area and so on. Thus, it is possible to induce current oscillations in this structure.To observe the desired oscillatory response we have to assume that both electrons and holes are capable of having ionizing collisions. They also showed the possibility of reducing the initial partial differential equations to coupled nonlinear maps. This allowed them to investigate the stability of the steady-state, periodic and chaotic motion and to study the application of a feedback signal to control this device.

In this work we study the behavior of a device consisting of two consecutive p-n-junctions connected via an intrinsic semiconductor under reverse biased voltage. We are interested in finding the conditions under which such a device will give an oscillatory-like response under the passage of a high energy particle, such as a minimum ionizing particle (MIP). which will cause the initial impact ionization inside the structure. This guarantees the $\delta$-like distribution as initial condition, since a MIP does not normally generates a charged shower. We present an approach based on the solution of the initial and boundary value problem for linear hyperbolic partial differential equations with linear and nonlinear boundary conditions [11]-[14]. This approach is common to describe the spatiotemporal dynamics of the charge and current densities in a p-n-i-p-n-structure. For different mobilities for the carriers the problem is reduced to a coupled nonlinear difference (DE) or difference-delay equations (DDE). We show that when all parameters for electrons and holes are equal the problem is reduced to two independent DE or DDE. We study numerically the evolution of the initial current density pulse (produced by high energy particle for instance) for different combinations of the device parameters. We show the possibility of oscillatory response to the passage of a high energy particle when the reverse bias voltage does not exceed the avalanche breakdown threshold, and the duration of the initial pulse does not exceed the time of flight of electrons and holes across the intrinsic region. This regime provides an internal amplification of the initial pulse of current produced by the particle. Finally the problem of stability of the


Fig. 1. Schematic view of a p-n-i-p-n-device and internal E-field distribution.
equilibrium state of a particle detector based on this principle is discussed.

## II. The Model

Let us consider a semiconductor structure, consisting of two p-n-junctions, connected through an intrinsic semiconductor and brought close to, rather than exceed, the avalanche breakdown threshold via reverse biased voltage. Two depleted slabs with a low conductivity are formed in the neighborhood of these junctions due to the reverse biased voltage applied. The conductivity of the intrinsic region is much larger than that of the depleted regions and at the same time, much less than the conductivity of the $\mathbf{p}$ - and $\mathbf{n}$-layers. Therefore, the distribution of the electric field strength across this structure has two peaks around the physical p-n-junctions, as shown in Fig. 1. The dynamics of the charge and current densities and $\mathbf{E}$-field in classical semiconductor devices is governed by a system of self-consistent equations, consisting of the full current, the discontinuity and Poison's equations (see for example [15]).

In the case of stabilized voltage applied across the junctions the Cauchy problem should be solved for the whole structure, since it is not possible to fix the currents at the ends. Due to the different properties of the n -, p -, and i-regions we have to use different sets of equations for different regions. This means that we have to match solutions obtained for two adjoining regions according to the corresponding boundary conditions at these borders. Therefore we need to solve the initial-boundaryvalue problem for each of the three regions, matching the solutions at the boundaries. The boundary conditions depend on the process that takes place in the device. In a general case we need to use numerical simulations to obtain the solution of the above mentioned system of basic equations with corresponding boundary and initial conditions. Taking into account the qualitative picture of the oscillatory regime described above and making some additional assumptions, we can further simplify the problem, achieving in this way a description of the regime studied.

Consider a 1-D case, and assume the semiconductor to be uniformly doped, and the number of electron-hole pairs generated by impact ionization to be small compared to the total number of pairs present in the junction. Under these conditions the internal electric field is not sensibly changed. Therefore the self consistency of the E-field is not needed, and the Poisson equation can be neglected.

Under the conditions of reverse biasing the diffusion current can also be neglected compared to the drift current, as well as the charge particle recombination process. The system of equations is reduced to:

$$
\begin{align*}
& \frac{1}{v_{n}} \frac{\partial j_{n}(x, t)}{\partial t}+\frac{\partial j_{n}(x, t)}{\partial x}=\alpha_{n} j_{n}(x, t)+\alpha_{p} j_{p}(x, t) \\
& \frac{1}{v_{p}} \frac{\partial j_{p}(x, t)}{\partial t}-\frac{\partial j_{p}(x, t)}{\partial x}=\alpha_{n} j_{n}(x, t)+\alpha_{p} j_{p}(x, t) \tag{1}
\end{align*}
$$

inside the slabs and

$$
\begin{align*}
& \frac{1}{v_{n}} \frac{\partial j_{n}(x, t)}{\partial t}+\frac{\partial j_{n}(x, t)}{\partial x}=0 \\
& \frac{1}{v_{p}} \frac{\partial j_{p}(x, t)}{\partial t}-\frac{\partial j_{p}(x, t)}{\partial x}=0 \tag{2}
\end{align*}
$$

outside the slabs. Here $j_{n}(x, t)\left(j_{p}(x, t)\right), v_{n}\left(v_{p}\right)$, and $\alpha_{n}\left(\alpha_{p}\right)$ denote the current density, velocity and avalanche amplification rate respectively for electrons (holes). The currents refer to drift currents only.

To formulate the boundary conditions let us consider the three regions of the p-n-i-p-n-structure shown in Fig. 1: two depletion slabs 1 and 3 and a drift region 2 (i-region). In the slab 1 holes are moving toward the left, while at the right end $(x=l)$ the holes are coming continuously inside slab 1 from the i-region. At the same time the electrons are moving toward the right end and due to reverse bias voltage, there is no incoming electrons (we neglect the current of minority carriers). For slab 1 we write the following boundary conditions:

$$
\begin{equation*}
j_{p}^{(1)}(l, t)=\mathbf{j}_{\mathbf{p}}^{(2)}(1, \mathbf{t}) \text { and } j_{n}^{(1)}(-l, t)=0 \tag{3}
\end{equation*}
$$

Analogously, for the slab 3

$$
\begin{equation*}
j_{n}^{(3)}(L-l, t)=\mathbf{j}_{\mathbf{n}}^{(2)}(\mathbf{L}-\mathbf{l}, \mathbf{t}) \text { and } j_{p}^{(3)}(L+l, t)=0 \tag{4}
\end{equation*}
$$

where the subscripts 1,2 and 3 refer to the slabs.
For the i-region the boundary conditions are:

$$
\begin{equation*}
j_{n}^{(2)}(l, t)=\mathbf{j}_{\mathbf{n}}^{(\mathbf{1})}(\mathbf{l}, \mathbf{t}) \text { and } j_{p}^{(2)}(L-l, t)=\mathbf{j}_{\mathrm{p}}^{(\mathbf{3})}(\mathbf{L}-\mathbf{l}, \mathbf{t}) . \tag{5}
\end{equation*}
$$

Here $\mathbb{L}$ is the width of the intrinsic region. Under these assumptions the self consistent description of the structure can be separated into two independent problems. One of them describes the process of the charged particles avalanche multiplication inside the depletion slabs with initial and boundary conditions given by (3) or (4). The second describes the free propagation of electrons and holes inside i-region and hence the interaction of the two slabs.

We can further simplify the problem assuming that the width of the depleted slabs is much less than the distance between them. We can then treat this process as a sequence of the
resulting transformations of the electron (hole) cloud into the hole (electron) cloud on the slabs.

## III. Phenomenological Boundary Conditions

From (1) we can find the dependence on the incoming current of the avalanche current output from each slab. Then, we use these values of the currents at the edges of the slabs as boundary conditions for (2) to calculate the currents inside the i-region. Therefore, we can formulate the initial- and boundary-value problem for the linear hyperbolic equations (2) with nonlinear (in general case) boundary conditions and use the method of characteristics to solve it [11]-[14]. This method allows us to reduce the initial-boundary-value problem for hyperbolic partial differential equations to the initial-value problem for ordinary difference or differencedifferential equations. The spatio-temporal dynamics of the charge and current densities is easily obtained. Moreover, if we are not interested in the details of the processes inside the slabs, we can use phenomenological boundary conditions of a different kind to investigate the possibility of an oscillatory regime in this device.

For simplicity, we consider the mobilities of electrons and holes to be constant. So, we can rewrite (2) as follows

$$
\begin{align*}
& \frac{1}{\nu_{n}} \frac{\partial j_{n}(\varsigma, \tau)}{\partial \tau}+\frac{\partial j_{n}(\varsigma, \tau)}{\partial \varsigma}=0 \\
& \frac{1}{\nu_{p}} \frac{\partial j_{p}(\varsigma, \tau)}{\partial \tau}-\frac{\partial j_{p}(\varsigma, \tau)}{\partial \varsigma}=0 \tag{6}
\end{align*}
$$

where $\tau=t\left(v_{n}+v_{p}\right) / 2 L$ and $\varsigma=x / L$ are dimensionless time and space variables; $\nu_{n}=2 v_{n} /\left(v_{n}+v_{p}\right)$ and $\nu_{p}=2 v_{p} /\left(v_{n}+v_{p}\right)$ are dimensionless normalized drift velocities of electrons and holes respectively and $L$ is the distance between the centers of the slabs. To these equations we should add the boundary and initial conditions for the $j_{n}(\varsigma, \tau)$ and $j_{p}(\varsigma, \tau)$.

In general, the transformations of the electron and hole clouds on the slabs are nonlinear and depend not only on the current densities of incoming clouds, but also on their derivatives taken at the boundaries of the slab. In the system under consideration there are two characteristic times: $-t_{a}$, the time of the avalanche multiplication developing (delay) and $-T_{f}$, the time of flight of the charge carriers through the intrinsic band of the structure. If $t_{a} \ll T_{f}$, we can neglect the transients in the slabs during avalanche multiplication as well as the delay of the avalanche current and hence, the dependence of the outcoming current on the derivatives. If $t_{a} \gg T_{f}$, this dependence on the derivatives becomes important and cannot be neglected.

Let us consider first the case when $t_{a} \ll T_{f}$. The boundary conditions are:

$$
\begin{align*}
& j_{p}(0, \tau)=f\left[j_{n}(0, \tau)\right]  \tag{7}\\
& j_{n}(1, \tau)=g\left[j_{p}(1, \tau)\right] \tag{8}
\end{align*}
$$

while the initial conditions have the form

$$
\begin{align*}
& j_{n}(\varsigma, 0)=j_{0 n}(\tau): \tau \in\left[0, \tau_{n}\right)  \tag{9}\\
& j_{p}(\varsigma, 0)=j_{0 p}(\tau): \tau \in\left[\tau_{n}, \tau_{p}\right) \tag{10}
\end{align*}
$$

where $\tau_{n}=1 / \nu_{n}$ and $\tau_{p}=1 / \nu_{p}$ are the dimensionless electrons and holes times of flight through i-region, respectively, and $j_{0 n}(\tau)$ and $j_{0 p}(\tau)$ the initial distributions of electron and hole current densities across the i-region, respectively. In (9) the initial distribution dependence on the space coordinate is transformed into the dependence on time according to the relationships $z= \pm v t$, which are valid for the beginning of the process.

The perturbation, caused by an external source (such as the passage of a high energy particle through the structure) defines the initial distribution of the current densities and can be approximately represented as follows:

$$
\begin{equation*}
j_{0 n}(\tau)=J_{0 n} \exp \left\{-\frac{\tau^{2}}{\delta \tau^{2}}\right\} \text { and } j_{0 p}(\tau)=0: \tau \in\left[0, \tau_{n}\right) \tag{11}
\end{equation*}
$$

if the particle is coming from the left and

$$
\begin{equation*}
j_{0 p}(\tau)=J_{0 p} \exp \left\{-\frac{\tau^{2}}{\delta \tau^{2}}\right\} \text { and } j_{0 n}(\tau)=0: \tau \in\left[\tau_{n}, \tau_{p}\right) \tag{12}
\end{equation*}
$$

if the particle is coming from the right.
Here $\delta \tau$ is the dimensionless duration of the initial pulse of current, assumed to be very small $\delta \tau \ll 1$ and $J_{0 n}, J_{0 p}$ the amplitudes of the initial pulse of current.

We define the evolution of this perturbation in time and space according to (6) and boundary conditions (7). It can be done using the property that an arbitrary solution of (6) is constant on the corresponding characteristics: $t-x / v_{p}=$ Const $_{1}$ and $t+x / v_{n}=$ Const $_{2}$. Using the method of characteristics [11]-[14] the initial-boundary-value problem defined by (6), (7), and (11) (or (12)) is reduced to the following system of two nonlinear difference equations:

$$
\begin{align*}
j_{p}(0, \tau) & =f\left[j_{n}\left(1, \tau-\tau_{p}\right)\right]  \tag{13}\\
j_{n}\left(1, \tau-\tau_{p}\right) & =g\left[j_{p}\left(0, \tau-\tau_{p}-\tau_{n}\right)\right] \tag{14}
\end{align*}
$$

with initial conditions (11) or (12). Thus, the solution of this problem depends on the initial distribution of the current density functions $j_{0 n}(\varsigma, 0)$ and $j_{0 p}(\varsigma, 0)$, as well as on the functions $f(x)$ and $g(x)$, describing, in general, nonlinear transformations of the density of electrons and holes clouds at the depleted slabs, i.e. at the ends of the intrinsic band. These functions are determined by the laws of impact ionization in the depleted slabs of this structure.

The physical nature of the impact ionization allows us to set the amount of generated electron-hole pairs proportional to the initial amount of particles which starts the avalanche multiplication. This dependence can be assumed at least in the regime of small charge densities. But, with the growing of the incoming cloud density we can reach the situation where all atoms will be completely ionized. This means that if the number of incoming charge particles increases the output current density will not be changed and saturation will take place. This situation is typical for weak doped semiconductors. On the other hand, for strongly doped samples the density of the generated charge carriers could become so large that it
will start to affect the value of the $\mathbf{E}$-field inside the slabs, changing its spatial distribution across the junctions.

Taking into account these properties of the impact ionization process we can consider three qualitatively different cases to check the possibilities of the existance of the oscillatory regime. Namely:
a) linear dependence of the generated carriers density on the density of the incoming charges

$$
\begin{equation*}
j_{p}(0, t)=M_{n} * j_{n}(0, t) \text { and } j_{n}(L, t)=M_{p} * j_{p}(L, t) \tag{15}
\end{equation*}
$$

where $M_{n}$ and $M_{p}$ are the electron and hole multiplication rates, defined by rate of doping, $\mathbf{E}$-field, sample materials etc;
b) linear increasing followed by saturation at a given value of the density of incoming charges, approximated by the hyperbolic tangent

$$
\begin{equation*}
j_{p}(0, t)=M_{p} * \tanh \left[j_{n}(0, t)\right] \tag{16}
\end{equation*}
$$

and

$$
j_{n}(L, t)=M_{n} * \tanh \left[j_{p}(L, t)\right]
$$

c) and finally, increasing, saturation and decreasing as a function of the growth of the incoming charge cloud density. In this case we could use the logistic function for $f$ and $g$

$$
\begin{equation*}
j_{p}(0, t)=A * j_{n}(0, t)\left[1-j_{n}(0, t)\right] \tag{17}
\end{equation*}
$$

and

$$
\begin{equation*}
j_{n}(L, t)=B * j_{p}(L, t)\left[1-j_{p}(L, t)\right] \tag{18}
\end{equation*}
$$

When the avalanche developing time $\delta_{p}, \delta_{n}$ is comparable to the charge carriers time of flight through the i-region, we need to take into account the dependencies of the outcoming currents on the derivatives. The simplest way to do it is to use the following expressions [14]

$$
\begin{align*}
\delta_{p} \frac{d j_{p}(0, t)}{d t}+j_{p}(0, t) & =f\left[j_{n}(0, t)\right] \\
\delta_{n} \frac{d j_{n}(L, t)}{d t}+j_{n}(L, t) & =g\left[j_{p}(L, t)\right] \tag{19}
\end{align*}
$$

where $\delta_{n}, \delta_{p}$ are characteristic times, defined by the time of developing of avalanche multiplication and depend on the slab width, the avalanche rates, E-field strength, rate of doping, mobilities of charge carriers etc.

Using these conditions, instead of (7) we obtain two difference-differential equations with delay time

$$
\begin{align*}
\delta_{p} \frac{d j_{p}(0, \tau)}{d \tau}+j_{p}(0, \tau) & =f\left[j_{n}\left(1, \tau-\tau_{n}\right)\right] \\
\delta_{n} \frac{d j_{n}\left(1, \tau-\tau_{n}\right)}{d \tau}+j_{n}\left(1, \tau-\tau_{n}\right) & =g\left[j_{p}\left(0, \tau-\tau_{n}-\tau_{p}\right)\right] \tag{20}
\end{align*}
$$



Fig. 2. Current oscillations formed by hole and electron pulses amplified in the depleted slabs due to avalanche multiplication. Delay time of avalanche equals to zero.

## IV. EQUAL p-n-Junctions

When both junctions as well as propagation conditions for electrons and holes inside the intrinsic region are equal, the systems of (13) and (20) are reduced to a single DD and DDE respectively. In this case we set $v_{n}=v_{p}=v, f=g$, $\tau_{n}=\tau_{p}=\tau_{0} j_{n}=j_{p}=j$ and $\delta_{n}=\delta_{p}=\delta_{0}$. Therefore, (13) and (20) are transformed into the form

$$
\begin{equation*}
j(0, \tau)=f\left[f\left[j\left(0, \tau-2 \tau_{0}\right)\right]\right] \tag{21}
\end{equation*}
$$

and

$$
\begin{equation*}
2 \delta_{0} \frac{d j(0, \tau)}{d \tau}+j(0, \tau)=f\left[f\left[j\left(0, \tau-2 \tau_{0}\right)\right]\right] . \tag{22}
\end{equation*}
$$

Similar equations have been obtained in [12]-[14] for the spatio-temporal dynamics of electromagnetic fields in the 1-D resonator with nonlinear reflecting walls.

In the case of linear multiplication of charges (see (15)) we have the following linear DE and DDE

$$
\begin{equation*}
j(0, \tau)=M^{2} j\left(0, \tau-2 \tau_{0}\right) \tag{23}
\end{equation*}
$$

and

$$
\begin{equation*}
2 \delta_{0} \frac{d j(0, \tau)}{d \tau}+j(0, \tau)=M^{2} j\left(0, \tau-2 \tau_{0}\right) \tag{24}
\end{equation*}
$$

The solution of (23) is proportional to the exponential function with exponent $\alpha=\frac{\operatorname{Ln}(M)}{\tau_{0}}$. Fig. 2 illustrates this solution, showing the exponential growth of the initial pulse of current where the period $T$ of the sequence of pulses is defined by the width of the intrinsic layer and the drift velocities, $T=L / v_{n, p}$. The main frequency of the signal of the external circuit is $f=v / L$. Note that the initial pulse conserves its shape.

The delay in the avalanche multiplication leads to a permanent growth of the width of the current pulses with time (see Fig. 3). If under this condition the amplitude of the pulses reaches the saturation level of the multiplication function given by (16), then the amplitude of the current pulses ceases to, and at the same time thier width continue to grow (see Fig. 4). The rather large value of the avalanche delay time $\tau_{0}$ destroys the oscillatory response (Fig. 5) because the avalanche multiplication inside a given depleted slab continues such a


Fig. 3. Effect of the delay of avalanche multiplication in the depleted slabs ( $\nu=0.033$ ). Delay, gain and extension of the duration of the pulses take place.


Fig. 4. Saturation of the internal gain. Duration of the output pulses extends due to delay of the avalanche multiplication inside the depleted slabs


Fig. 5. Transformation of the oscillatory regime of the detector to DC current regime due to nonzero delay of avalanche multiplication.
long time that the first charges generated to have time to reach the next depleted slab, thus producing a new avalanche.

Hence, qualitatively the main characteristics of oscillatory regime remain in the case of unequal mobilities, but some parameters of oscillations are different.

Namely: 1) the phenomenon of shortening or lengthening the duration of a pulse is correlated respectively to the decreasing or increasing of the charges drift velocity; 2 ) the main oscillation frequency, defined by $f=\left(v_{p} v_{n}\right) / L\left(v_{p}+v_{n}\right)$ will decrease, because the period of these oscillations is reciprocal to the sum of times of flight of holes and electrons through the intrinsic region; 3) when the mobilities of the holes and


Fig. 6. Current oscillations for the case of different mobilities of the holes and electrons $v_{e} / v_{p}=1 / 5$. Delay time of the avalanche multiplication equals zero.


Fig. 7. Destruction of oscillations for nonzero delay of avalanche multiplication for the case of unequal mobilities of the electrons and holes.
electrons are very different the waveform of outside circuit signal will look like a series of double pulses: one of them produced by the electron current and the other by the hole current. This can be useful to detect higher harmonics. Fig. 6 and Fig. 7 illustrate this case.

## V. CONCLUSIONS

We have presented a simplified model of a device to detect the passage of high energy particles and have shown the existence of an oscillatory response of the current inside p-n-i-p-n-junction, of period much shorter than the mean distance between events.

Concerning the stability of the equilibrium state of this device, we notice from the solution of (23) that the internal gain of current fluctuations always exists for the multiplication rates $M_{n} M_{p}>1$. Therefore in an ideal case we should provide some current threshold, not allowing the internal avalanche gain of the current pulses, which is thermally generated. Otherwise, this will lead to self-excitation of the current oscillations, causing instabilities in the equilibrium state. To overcome this problem it is possible to set traps for holes between slabs [7], doping the intrinsic region to reduce their amount to a proper level. In this case the corresponding number of thermally or in any other way generated holes will be captured by the traps, eliminating the positive feedback and preventing the developing of the self-oscillations. If the number of initial holes exceeds the number of the traps, then
uncaptured holes will reach the left slab and thus initiate the avalanche multiplication of the useful signal.

On the other hand, this instability of the current inside the structure with respect to fluctuations could be used to develop a new semiconductor oscillator of high frequency. The material should be such as to provide the conditions for existence of the nonlinear characteristic of the third kind described in Section 3, i.e. the logistic-like dependence of the outcoming currents on the incoming. And as it follows from the results of [12] and [14], the chaotic oscillations of the electrons and holes currents could be generated in such a device for high enough rate of avalanche multiplication.

We suggest these devices be considered for construction in order to check these models, since present technology allows it.

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Konstantin A. Lukin was born in Dushanbe, Tadjikistan, USSR. He received the Diploma-Engineer degree in 1973 from Kharkov State University, Ukraine. Since 1973, he has been with the Institute of Radiophysics and Electronics, National Academy of Sciences, Kharkov. He received the Candidate of Sciences (physics and mathematics) degree in 1980 from Moscow State University, and the Doctor of Sciences degree in 1989 from Kharkov State University. His dissertations were devoted to theory of nonlinear processes in microwave and millimeter wave elec5tronic devices. He is author of more than 60 journal publications in this field. His current research interests are nonlinear dynamics of different physical systems, particularly systems with delay feedback, as well as generation of chaotic oscillations and noise signal processing.


Hilda A. Cerdeira was born in Argentina. She graduated from the Universidad de Buenos Aires and received the Ph.D. degree from Brown University, Providence, RI, in 1972. She worked at the Max Planck Institut für Festköperforschung, Stuttgart, Germany, and at the Universidade Estadual de Campinas, Brazil. She is currently with the International Centre for Theoretical Physics, Condensed Matter Group, Trieste, Italy, where she is in charge of the Adriatico Research Conferences. Her main field of research is nonlinear dynamics, with special emphasis of the synchronization of systems with a large number of elements.


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    K. A. Lukin is with the Institute of Radiophysics and Electronics, National Academy of Sciences of Ukraine, Kharkov 310085, Ukraine.
    H. A. Cerdeira and A. A. Colavita are with the International Centre for Theoretical Physics, Trieste 34100, Italy

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[^1]:    Alberto A. Colavita was born in Argentina. He graduated in physics from the Universidad Nacional de Cuyo and received the Ph.D. degree from Washington University, St. Louis, MO, in 1974. He taught physics for nearly 20 years and is now full Professor of Computer Architecture, Universidad Nacional de San Luis and Director of the Microprocessor Laboratory, International Centre for Theoretical Physics, Trieste, Italy. He pioneered and is very active in the design, using silicon microstrip diodes, of the next generation of gamma ray telescopes.

