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Surface Impedance of Clean Type II Superconductors

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The surface impedance of clean type II superconductors is calculated in fields near H_{c2}. *A marked dependence of the surface impedance on mean free path* l, *is predicted which is consistent with experiment. The dependence of the surface impedance on the direction of propagation of the microwave is also discussed.*

1. INTRODUCTION

Recently we have used a theory, built around the Green's function of Brandt *et al.*,[1] to calculate several transport coefficients of clean type II superconductors in the vicinity of the upper critical field H_{c2}.[2–4] The most important prediction of the theory was the strong mean free path dependence of the transport coefficients, which was consistent with the ultrasonic attenuation measurements of Forgan and Gough.[5] Since that time measurements of the attenuation[6,7] and thermal conductivity[8] have been made over a wide field range below H_{c2} for samples of different purity. Both mean free path and magnetic field dependence are in good qualitative agreement with theory.

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In order to obtain further comparison with experiment, and hopefully to stimulate further work in this area, we present in this paper theoretical predictions of the dependence of the microwave surface impedance on mean free path, field, and direction of propagation of the microwave. This important parameter has received relatively little attenuation since the early work of Hibler and Maxfield.[9] The only theoretical work, by Hibler and Cyrot,[10] is in the infinite mean free path limit.

2. SURFACE IMPEDANCE

The surface impedance is defined by:

$$Z = 4\pi E(0)/H(0) \tag{1}$$

where $E(0)$ and $H(0)$ are the electric and magnetic fields at the surface of the sample. Assuming specular reflection at the surface,* we have

$$Z(\omega) = \frac{2}{\pi}\int_0^\infty \frac{dq}{\sigma(\mathbf{q},\omega) + iq^2/4\pi\omega} \tag{2}$$

where $\sigma(\mathbf{q}, \omega)$ is the electrical conductivity and $\mathbf{q}$ and ω are the wave vector and frequency of the electromagnetic wave. If we assume that the conductivity behaves like q^{-1}, the extreme anomalous limit, the surface impedance can be written as

$$Z_s/Z_n = (\sigma_s/\sigma_n)^{-1/3} \tag{3}$$

where the subscripts s and n stand for superconducting and normal phases, respectively.

In this work we calculate the surface impedance for a clean type II superconductor in the presence of a magnetic field slightly smaller than H_{c2}. The quasiparticle contribution to the conductivity is calculated by the method described in detail in Ref. 2; the contribution from dynamical fluctuations of the order parameter is obtained by following the procedure, outlined by Caroli and Maki.[12] Of course, the fluctuations do not contribute to the conductivity for $\mathbf{J} \parallel \mathbf{B}$ and we will not discuss this geometry further. When $\mathbf{J} \perp \mathbf{B}$ the conductivity is purely real, since its imaginary part is exactly cancelled, to order Δ^2, by the contribution from the dynamical fluctuations of the order parameter. Further, in the anomalous limit, $ql > 1$, it can be shown that the contribution from the fluctuations of the vortex lattice to the real part of the conductivity is very small, of order Δ^2, and can be neglected, in contrast to the $ql < 1$ limit,[3,4] where the main contribution comes from the fluctuations of the vortex lattice. It remains, then, to determine the real

* The result for diffuse reflection is 8/9 of that for specular reflection.[11]

part of the quasiparticle contribution to the conductivity

$$\sigma_{\mu\mu}(\mathbf{q},\omega) = Q_{j_\mu j_\mu}(\mathbf{q},\omega)/-i\omega \tag{4}$$

The response function $Q_{j_\mu j_\mu}(\mathbf{q},\omega_r)$ is defined as

$$Q_{j_\mu j_\mu}(\mathbf{q},\omega_\nu) = -2T\left(\frac{e}{m}\right)^2 \sum_{\omega_n} \int \frac{d^3p}{(2\pi)^3}(p_\mu)^2(G,F) \tag{5}$$

where T is the temperature; e and m the charge and mass of the electron;

$$(G,F)(\mathbf{q},\omega_n-\omega_\nu,\omega_n) = G^{\mathrm{B}}_{\omega_n}(\mathbf{p})G^{\mathrm{B}}_{\omega_n-\omega_\nu}(\mathbf{p}-\mathbf{q}) \times\left[1+\Delta^2\int_{-\infty}^{\infty} d\alpha\, G^0_{-(\omega_n-\omega_\nu)}(\mathbf{p}-\mathbf{q};2\alpha)\rho_{00}(\alpha,\Omega)G^0_{-\omega_n}(\mathbf{p};2\alpha)\right] \tag{6}$$

where

$$\rho_{00}(\alpha,\Omega) = \frac{2}{\sqrt{\pi}}\frac{\exp[-(2\alpha/k_c v_{\mathrm{F}}\sin\theta)^2]}{k_c v_{\mathrm{F}}\sin\theta} \tag{7}$$

$$G^0_{\omega_n}(\mathbf{p};2\alpha) = [i\omega_n - \xi_P - 2\alpha]^{-1} \tag{8}$$

$$G^{\mathrm{B}}_{\omega_n}(\mathbf{p}) = \left[i\omega_n - \xi_P + i\sqrt{\pi}\frac{\Delta^2}{k_c v_{\mathrm{F}}\sin\theta}W\left(\frac{i\omega_n+\xi_P}{k_c v_{\mathrm{F}}\sin\theta}\right)\right]^{-1} \tag{9}$$

is the Green's function for a superconductor calculated by Brandt *et al.*[1]; and

$$W(z) = \frac{i}{\pi}\int_{-\infty}^{\infty} dt\frac{\exp(-t^2)}{z-t} \tag{10}$$

$k_c = (2eB)^{1/2}$; B is the magnetic induction; v_{F} is the Fermi velocity, $\xi_P = (p^2/2m) - \varepsilon_{\mathrm{F}}$, with ε_{F} as the Fermi energy; and θ denotes the angle between the direction of propagation of the quasiparticle and the static magnetic field. Carrying out the analytic continuation of $Q_{j_\mu j_\mu}(q,\omega_\nu)$ onto real frequencies, we obtain for the imaginary part of the response function:

$$\begin{aligned}\operatorname{Im} Q_{j_\mu j_\mu}(\mathbf{q},\omega_0) = &-\left(\frac{e}{m}\right)^2\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\left(\tanh\frac{\omega}{2T}-\tanh\frac{\omega-\omega_0}{2T}\right)\\ &\times\int\frac{d^3p}{(2\pi)^3}(p_\mu)^2\operatorname{Re}[(G,F)(\mathbf{q},\omega-\omega_0-i/2\tau,\omega+i/2\tau)\\ &-(G,F)(\mathbf{q},\omega-\omega_0-i/2\tau,\omega-i/2\tau)]\end{aligned} \tag{11}$$

Since all experiments in clean Nb show very little temperature dependence below 4 K, we only quote results at $T = 0$, where the real part of the conductivity is given by

$$\sigma_{\mu\mu}(\mathbf{q}, \omega, J \perp B) = \frac{e^2 m^2}{2\pi q} \int_0^{2\pi} \frac{d\phi}{2\pi} v_\mu^2 \left[1 + 2K^2 \left(\frac{ix_0}{\sin\theta}\right) \frac{k_c l r}{1 + k_c l r}\right] \tag{12}$$

the integral over ϕ is taken in the plane perpendicular to q, l is the electronic mean free path,

$$K(z) = [1 - i\sqrt{\pi}\,(\Delta/k_c v_F \sin\theta)^2 W'(z)]^{-1} \tag{13}$$

$$x_0 = (1/k_c l) + r \tag{14}$$

$$r = \frac{2(\Delta/k_c v_F)^2}{\alpha(x_0)\sin^2\theta + 4(\Delta/k_c v_F)^2} \times \left\{\left[\frac{1}{(k_c l)^2} + \alpha \sin^2\theta + 4\left(\frac{\Delta}{k_c v_F}\right)^2\right]^{1/2} - \frac{1}{k_c l}\right\} \tag{15}$$

and $2 > \alpha(x_0) > 4/\pi$.

We distinguish now between the geometries $\mathbf{q}\|\mathbf{B}$ and $\mathbf{q}\perp\mathbf{B}$. For propagation parallel to the external field the surface resistance is given by

$$\frac{R_s}{R_n}(\mathbf{J} \perp \mathbf{B}; q\|\mathbf{B}) = \left(1 + \frac{2\mu}{1 + \mu}\right)^{-1/3} \tag{16}$$

while for propagation perpendicular to $\mathbf{B}$

$$\frac{\sigma_s}{\sigma_n} = 1 + \frac{3}{\pi}\int d\phi \sin^2\phi \frac{k_c l r}{1 + k_c l r} \simeq 1 + \frac{3}{\pi}\int d\phi \frac{\mu \sin^2\phi}{\mu + \sin\phi} \tag{17}$$

and

$$\frac{R_s}{R_n}(\mathbf{q} \perp \mathbf{B}; \mathbf{J} \perp \mathbf{B}) = \left[1 + \frac{12\mu}{\pi}\left(1 - \frac{\mu\pi}{2} + \frac{2\mu^2}{(\mu^2 - 1)^{1/2}} \tan^{-1}\left(\frac{\mu - 1}{\mu + 1}\right)^{1/2}\right)\right]^{-1/3} \tag{18}$$

where $\mu = \sqrt{\pi}(\Delta/k_c v_F)^2 k_c l$, differs by a factor of two from a similar parameter used in Refs. 3 and 4.

3. RESULTS AND DISCUSSION

Equations (16) and (17) predict a very strong mean free path dependence which is consistent, qualitatively, with the experimental results of Hibler and Maxfield.[9] These authors also observed a 5% increase in R_s as the angle between the direction of propagation of the microwave and the magnetic field is increased from 0 to $\pi/2$. This result does not agree with the theory, which gives a value of $R_s(\mathbf{q} \perp \mathbf{B})/R_s(\mathbf{q} \| \mathbf{B})$ less than one. In order to determine whether this discrepancy is due to failure of the model or to experimental error, more measurements of these parameters are needed.

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