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Exact solution of the generalized time-dependent Jaynes–Cummings Hamiltonian

J.L. Gruver [1], J. Aliaga [2], Hilda A. Cerdeira [3] and A.N. Proto [2]
International Centre for Theoretical Physics, P.O. Box 586, 34100 Trieste, Italy

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A time-dependent generalization of the Jaynes–Cummings Hamiltonian is studied using the maximum entropy formalism. The approach, related to a semi-Lie algebra, allows one to find three different sets of physically relevant operators which describe the dynamics of the system for any temporal dependence. It is shown how the initial conditions of the operators are determined via the maximum entropy principle density operator, where the inclusion of the temperature turns the description of the problem into a thermodynamical one. The generalized time-independent Jaynes–Cummings Hamiltonian is exactly solved as a particular example.

Recently [1], it was shown that the dynamics and thermodynamics of a two-level system coupled to a classical field can be fully described in the frame of the maximum entropy principle (MEP) formalism and group-theory based methods [2]. When the field is quantized and only the difference between the level populations is of interest, the quantum two-level system leads to the well known two-level Jaynes–Cummings Hamiltonian (JCH) [3], which is one of the most used models in fields like quantum optics, NMR, quantum electronics, and matter–field interactions (e.g. to study periodic spontaneous collapse and revival). In the rotating wave approximation (RWA), this Hamiltonian becomes solvable and it has been broadly used in the last years [4–9].

The knowledge of the mean values of the field populations, correlation functions and nth-order coherence functions are of main interest in the usual applications. The MEP formalism allows us to describe a Hamiltonian system in terms of those, and only those, quantum operators *relevant* to the problem at hand. Thus, this formalism is suitable to study the Hamiltonian here considered. Since in our generalization the population of each level and not their difference is considered, the resulting Hamiltonian will be called a *generalized* time-dependent JCH.

The aims of this Letter are: (a) to show that the *generalized time-dependent* JCH is related to different sets of physically relevant operators depending on the initial information of the system, (b) to give the evolution equations for one of these sets for the *generalized time-dependent* JCH, (c) to evaluate the initial conditions through the maximum entropy density operator, which includes the temperature, resulting in a thermodynamical treatment of the problem at hand, and (d) to obtain the exact dynamical evolution for the *generalized time-independent* JCH.

In order to make clear the fundamental features of our approach we begin by summarizing the principal concepts of the MEP [10,11]. Given the expectation values $\langle \hat{O}_j \rangle$ of the operators $\hat{O}_j$, the statistical operator $\hat{\rho}(t)$ is defined by

$$\hat{\rho}(t)=\exp\left(-\lambda_0\hat{I}-\sum_{j=1}^{L}\lambda_j\hat{O}_j\right), \qquad (1)$$

[1] On leave from Grupo de Sistemas Dinámicos, Universidad de Buenos Aires – Centro Regional Norte, C.C. 2, (1638) V. López, Buenos Aires, Argentina.

[2] Permanent address: Grupo de Sistemas Dinámicos, Universidad de Buenos Aires – Centro Regional Norte, C.C. 2, (1638) V. López, Buenos Aires, Argentina.

[3] Also at Universidade Estadual de Campinas, C.P. 6165, 13081 Campinas, São Paulo, Brazil.

where L is a natural number or infinity, and the $L+1$ Lagrange multipliers λ_j are determined to fulfill the set of constraints

$$\langle \hat{O}_j \rangle = \mathrm{Tr}[\hat{\rho}(t)\hat{O}_j] , \quad j=0, 1, ..., L \tag{2}$$

($\hat{O}_0 = \hat{I}$ is the identity operator) and normalization in order to maximize the entropy, defined (in units of the Boltzmann constant) by

$$S(\hat{\rho}) = -\mathrm{Tr}[\hat{\rho} \ln \hat{\rho}] . \tag{3}$$

The time evolution of the statistical operator is given by

$$i\hbar \frac{d\hat{\rho}}{dt} = [\hat{H}(t), \hat{\rho}(t)] . \tag{4}$$

One should endeavor to find the relevant operators entering eq. (1) so as to guarantee not only that S is maximum, but also is a constant of motion. Introducing the natural logarithm of eq. (1) into eq. (4) it can be easily verified that the *relevant operators* are those that close a semi-Lie algebra under commutation with the Hamiltonian, $\hat{H}$, i.e.

$$[\hat{H}(t), \hat{O}_j] = i\hbar \sum_{i=0}^{L} g_{ij}(t)\hat{O}_i . \tag{5}$$

Equation (5) defines an $L \times L$ matrix G and constitutes the central requirement to be fulfilled by the operators entering in the density matrix. The Liouville equation (4) can be replaced by a set of coupled equations for the mean values of the relevant operators or the Lagrange multipliers as follows [12],

$$\frac{d\langle \hat{O}_j \rangle_t}{dt} = -\sum_{i=0}^{L} g_{ij} \langle \hat{O}_i \rangle , \quad j=0, 1, ..., L , \tag{6}$$

$$\frac{d\lambda_j}{dt} = \sum_{i=0}^{L} \lambda_i g_{ji} , \quad j=0, 1, ..., L . \tag{7}$$

In the MEP formalism, the mean values of the operators and the Lagrange multipliers belong to dual spaces which are related by [11]

$$\langle \hat{O}_j \rangle = -\frac{\partial \lambda_0}{\partial \lambda_j} . \tag{8}$$

The *generalized time-dependent* JCH in the RWA takes the form

$$\hat{H} = E_1 \hat{b}_1^\dagger \hat{b}_1 + E_2 \hat{b}_2^\dagger \hat{b}_2 + \omega \hat{a}^\dagger \hat{a} + T(t)(\gamma \hat{a} \hat{b}_1 \hat{b}_2^\dagger + \gamma^* \hat{b}_2 \hat{b}_1^\dagger \hat{a}^\dagger) \tag{9}$$

($\hbar = 1$), where γ is the coupling constant between the system and the external field, E_j and ω are the energies of the levels and the field, respectively, $\hat{a}^\dagger$, $\hat{a}$, are boson operators, $\hat{b}_j^\dagger$ and $\hat{b}_j$ are fermion operators and $T(t)$ is an arbitrary function of time. In the problem considered in this Letter it turns out that the relevant operators can be presented in *three different but equivalent forms*, each of them having different physical interpretations. These sets are connected via isomorphisms which allow us to go from one set to another. The advantage of this multiple representation comes from the fact that when partial information in any set is known, for instance the initial values only for some operators are known, it is possible to complete the missing information via the isomorphisms, if the complementary data in any other set is known (i.e. mixed initial conditions [11]). Thus, first considering the level population it can be found that some basic relevant operators satisfying eq. (5) are obtained. These fundamental operators, appearing in the three possible sets of relevant operators, are

$$\hat{N}_1 = \hat{b}_1^\dagger \hat{b}_1 , \tag{10}$$

$$\hat{N}_2 = \hat{b}_2^\dagger \hat{b}_2 , \tag{11}$$

$$\hat{\Delta} = \hat{a}^\dagger \hat{a} , \tag{12}$$

$$\hat{I} = \gamma \hat{a} \hat{b}_1 \hat{b}_2^\dagger + \gamma^* \hat{b}_2 \hat{b}_1^\dagger \hat{a}^\dagger , \tag{13}$$

$$\hat{F} = i(\gamma \hat{a} \hat{b}_1 \hat{b}_2^\dagger - \gamma^* \hat{b}_2 \hat{b}_1^\dagger \hat{a}^\dagger) , \tag{14}$$

$$\hat{N}_{2,1} = \hat{b}_2^\dagger \hat{b}_2 \hat{b}_1^\dagger \hat{b}_1 . \tag{15}$$

$\hat{N}_l$, $l=1, 2$, and $\hat{\Delta}$ can be thought of as the population number of the levels and the external field, respectively. $\hat{I}$ can be considered as the interaction energy between the levels and the external field, $\hat{F}$ as the particle current between levels and, finally, $\hat{N}_{2,1}$ as the double occupation number. It is interesting to mention that the operators (10), (11), (13), (14) can be considered as the quantum counterpart of the operators obtained for the semiclassical two-level system studied in ref. [1]. One of the sets which closes a semi-Lie algebra with the Hamiltonian (see eq. (5)) reads

$$\hat{N}_1^n = (\hat{a}^\dagger)^n \hat{N}_1 (\hat{a})^n , \tag{16}$$

$$\hat{N}_2^n = (\hat{a}^\dagger)^n \hat{N}_2 (\hat{a})^n , \tag{17}$$

$$\hat{\Delta}^n = (\hat{a}^\dagger)^n \hat{\Delta} (\hat{a})^n , \tag{18}$$

$$\hat{I}^n = (\hat{a}^\dagger)^n \hat{I}(\hat{a})^n , \qquad (19)$$

$$\hat{F}^n = (\hat{a}^\dagger)^n \hat{F}(\hat{a})^n , \qquad (20)$$

$$\hat{N}^n_{2,1} = (\hat{a}^\dagger)^n \hat{N}_{2,1}(\hat{a})^n , \qquad (21)$$

$n=0, 1, \ldots$. This set can have the following physical interpretation: We can consider the operators with $n>1$ as a measure of virtual transitions due to the absorption of more than one photon and then emission of the extra photons in a transition between the levels. This interpretation arises from the fact that the powers of $\hat{a}^\dagger$ and $\hat{a}$ represent successive creation and annihilation of photons in the field. For $n=0$ eqs. (16)–(21) reduce to the fundamental set (eqs. (10)–(15)). This set is suitable for numerical simulation because it provides the simplest form of the system of differential equations for the evolution of their mean values.

The words *physically relevant operators* have a deep meaning in our context. It can be shown that the RWA applied at the beginning introduces in a natural way the set of correlation functions. The commutation between the Hamiltonian and particle current between levels ($\hat{F}$) give, for instance, the correlation operator between the population number of the levels and the field ($\hat{N}^1_1$). If the same procedure were applied to the Hamiltonian without the RWA the resulting algebra of operators would not be the same. This difference comes from the fact that the generator of the algebra, the Hamiltonian, generates a set of operators which are closely related to the physics of the problem and in this sense we mean that the set is *physically* relevant.

It can be shown that eq. (5) is also fulfilled by the sets

$$\{\tfrac{1}{2}[\hat{O}_i(\hat{a}^\dagger)^n(\hat{a})^n + (\hat{a}^\dagger)^n(\hat{a})^n\hat{O}_i]\}^\infty_{n=0} ,$$

and

$$\{\tfrac{1}{2}[\hat{O}_i(\hat{a}^\dagger\hat{a})^n + (\hat{a}^\dagger\hat{a})^n\hat{O}_i]\}^\infty_{n=0} ,$$

where the $\hat{O}_i$ are the fundamental operators given by eqs. (10)–(15). The first set can be interpreted as the correlation functions between the fundamental operators and $(\hat{a}^\dagger)^n(\hat{a})^n$, which are proportional to the nth-order coherence function of the field (see for example ref. [13]). The operators included in the second one are proportional to the correlations between the fundamental operators and the energy of the field. Detailed calculations including the transformations between sets will be published elsewhere. From now onwards, and in order to study the dynamical and thermodynamical features of this system, we will deal with the set defined by eqs. (16)–(21).

The dynamical equations for the operators (16)–(21) can be obtained using the Ehrenfest theorem (eq. (6)), and are given by

$$\frac{\mathrm{d}\langle \hat{N}^n_1\rangle}{\mathrm{d}t} = T(t)\langle \hat{F}^n\rangle + nT(t)\langle \hat{F}^{n-1}\rangle , \qquad (22)$$

$$\frac{\mathrm{d}\langle \hat{N}^n_2\rangle}{\mathrm{d}t} = -T(t)\langle \hat{F}^n\rangle , \qquad (23)$$

$$\frac{\mathrm{d}\langle \hat{F}^n\rangle}{\mathrm{d}t} = -\alpha\langle \hat{I}^n\rangle + 2|\gamma|^2 T(t)[(n+1)\langle \hat{N}^n_2\rangle - \langle \hat{N}^{n+1}_1\rangle + \langle \hat{N}^{n+1}_2\rangle - (n+1)\langle \hat{N}^n_{2,1}\rangle] , \qquad (24)$$

$$\frac{\mathrm{d}\langle \hat{I}^n\rangle}{\mathrm{d}t} = \alpha\langle \hat{F}^n\rangle , \qquad (25)$$

$$\frac{\mathrm{d}\langle \hat{\Delta}^n\rangle}{\mathrm{d}t} = (n+1)T(t)\langle \hat{F}^n\rangle , \qquad (26)$$

$$\frac{\mathrm{d}\langle \hat{N}^n_{2,1}\rangle}{\mathrm{d}t} = 0 , \qquad (27)$$

$n=0, 1, \ldots$, where $\alpha = E_2 - E_1 - \omega$. Equations (22)–(27) are the exact dynamical evolution equations of the relevant operators for the *generalized time-dependent* JCH. They can be thought of as a kind of generalized Bloch equations for the quantum field case. As can be seen, the different order correlations are connected via the operators $\hat{N}^n_1$ and $\hat{F}^n$ (eqs. (22), (24)). It can be easily proved that

$$\{\langle(\hat{a}^\dagger)^n\hat{N}_1(\hat{a})^n\rangle + \langle(\hat{a}^\dagger)^n\hat{N}_2(\hat{a})^n\rangle - \langle(\hat{a}^\dagger)^{n-1}\Delta(\hat{a})^{n-1}\rangle\}^\infty_{n=0} , \qquad (28)$$

$$\{\langle(\hat{a}^\dagger)^n\hat{N}_{2,1}(\hat{a})^n\rangle\}^\infty_{n=0} , \qquad (29)$$

are constants of the motion. In particular the particle current between levels is equal to the photon flux. For $n=0$ we reobtain the conservation of the level population and for $n>0$ we obtain a restriction for the correlations. Equation (28) shows that the mean value of the operators will not be independent, giving a restriction on the choice of the initial conditions. Therefore, it is crucial to use a formalism that allows a proper evaluation of these initial conditions. In order to do this we will use the MEP density matrix given in eq. (1). In the MEP context, the initial conditions can be settled down in the dual space of Lagrange multipliers. These Lagrange multipliers are numbers that can be freely chosen, contrariwise the mean values cannot (see eq. (28)). The equivalent restrictions for the Lagrange multipliers are automatically settled when the density operator is diagonalized, as we will see below. We note that in our formalism lack of knowledge of the mean value of one operator is equivalent to setting its Lagrange multiplier equal to zero.

In order to derive a thermodynamical approach to the problem at hand, we will write the density matrix including the Hamiltonian as a relevant operator. Then, the statistical operator can be written as

$$\hat{\rho}(t)=\exp\left(-\lambda_0\hat{I}-\beta\hat{H}-\sum_{n=0}^{\infty}(\lambda_1^n\hat{N}_1^n+\lambda_2^n\hat{N}_2^n+\lambda_3^n\hat{F}^n+\lambda_4^n\hat{I}^n+\lambda_5^n\hat{N}_{2,1}^n+\lambda_6^n\hat{A}^n)\right). \qquad (30)$$

Diagonalizing and taking the trace of $\hat{\rho}(t)$ we arrive to the following expression for λ_0 in terms of the other Lagrange multipliers,

$$\lambda_0=\ln\left(\sum_{r=1}^{\infty}\exp(-K_{1,r})\times 2\cosh(K_{2,r})+\exp(-\beta E_1-\lambda_1^0)+\sum_{r=0}^{\infty}\exp(-K_{3,r})+\sum_{r=0}^{\infty}\exp(-K_{4,r})\right), \qquad (31)$$

where

$$K_{1,r}=\tfrac{1}{2}\beta[E_2+E_1+(2r-1)\omega]+\sum_{n=0}^{r}\left[\tfrac{1}{2}\lambda_1^n\Pi_r^{n-1}+\tfrac{1}{2}[\lambda_2^n+(2r-n-1)\lambda_6^n]\Pi_{r-1}^{n-1}\right], \qquad (32)$$

$$K_{2,r}=\sqrt{X_r^2+Y_r^2+Z_r^2}, \qquad (33)$$

$$K_{3,r}=\beta r\omega+\sum_{n=0}^{r}\lambda_6^n\Pi_r^n, \qquad (34)$$

$$K_{4,r}=\beta(E_2+E_1+r\omega)+\sum_{n=0}^{r}\Pi_r^{n-1}[\lambda_1^n+\lambda_2^n+\lambda_5^n+(r-n)\lambda_6^n], \qquad (35)$$

are *invariants of the motion* (this can be shown using eq. (7)),

$$X_r=\sqrt{r}\,|\gamma|\left(\beta T(t)+\sum_{n=0}^{r}\lambda_4^n\Pi_{r-1}^{n-1}\right), \qquad (36)$$

$$Y_r=\sqrt{r}\,|\gamma|\sum_{n=0}^{r}\lambda_3^n\Pi_{r-1}^{n-1}, \qquad (37)$$

$$Z_r=-\tfrac{1}{2}\beta\alpha+\sum_{n=0}^{r}\left\{\tfrac{1}{2}\lambda_1^n\Pi_r^{n-1}-\tfrac{1}{2}[\lambda_2^n-(n+1)\lambda_6^n]\Pi_{r-1}^{n-1}\right\}, \qquad (38)$$

and $\Pi_r^m\equiv\Pi_{j=0}^m(r-j)$, $\Pi_r^{-1}\equiv 1$. We want to mention that $\{X_r, Y_r, Z_r\}$ can be considered as a generalization of the vector model of the density matrix shown in ref. [13]. As in the case considered in ref. [13], one component of the vector is related to the level populations while the others are related to the real and imaginary parts of the non-diagonal elements of the density operator (interaction energy and particle current). In this sense we can consider $K_{2,r}$ as the norm of a vector in $\mathbb{R}^3$, with components $\{X_r, Y_r, Z_r\}$. So, this r-dependent sphere can be thought of as an extension in the dual space of Lagrange multipliers of a sort of quantized version of the Bloch sphere.

The initial conditions can be determined using eqs. (8) and (31). For example, the initial mean value of the population of the level 1 and its correlations with the field reads

$$\langle\hat{N}_1^n\rangle_0=\exp(-\lambda_0)\left(\exp(-\beta E_1-\lambda_1^0)\delta_{n,0}+\sum_{r=1}^{\infty}\Pi_r^{n-1}\{\exp(-K_{1,r})[\cosh(K_{2,r})-(Z_r/K_{2,r})\sinh(K_{2,r})]\}+\sum_{r=0}^{\infty}\exp(-K_{4,r})\right), \qquad (39)$$

δ is the Kronecker function. Thus, eq. (31) gives the exact thermodynamical solution for the *generalized time-dependent* JCH.

Now, in order to show how these ideas can be applied, we consider the Hamiltonian (9) in the time-independent case. Thus, the Hamiltonian reads

$$\hat{H}=E_1\hat{b}_1^\dagger\hat{b}_1+E_2\hat{b}_2^\dagger\hat{b}_2+\omega\hat{a}^\dagger\hat{a}+\gamma\hat{a}\hat{b}_1\hat{b}_2^\dagger+\gamma^*\hat{b}_2\hat{b}_1^\dagger\hat{a}^\dagger\,. \qquad (40)$$

For this case, eqs. (22)–(27) can be explicitly solved. If one introduces the quantized generalized Rabi flipping frequency [13],

$$\Omega_n^2\equiv\alpha^2+(n+1)\epsilon^2\equiv\Omega_0^2+n\epsilon^2\,,$$

$$\epsilon^2\equiv4|\gamma|^2\,,\quad \Theta_{nk}\equiv\Omega_n/\Omega_k\,,$$

the exact solution for $\langle\hat{N}_1^0\rangle_t$ is

$$\begin{aligned}\langle\hat{N}_1^0\rangle_t=&\langle\hat{N}_1^0\rangle_0\\&+\left(\frac{8\langle\hat{\Lambda}^0\rangle_0}{\Omega_0^2}-\frac{\epsilon^2\langle\hat{N}_2\rangle_0}{2\Omega_0^2}\right)C_0(t)+\frac{\langle\hat{F}\rangle_0}{\Omega_0}S_0(t)\\&+\sum_{n=1}^{\infty}\left(\frac{\langle\hat{\Lambda}^n\rangle_0}{\Omega_n^2}+\frac{\alpha^2\langle\hat{N}_2^n\rangle_0}{2\Omega_n^2}\right)\\&\times\sum_{k=0}^{n-1}b_{n,k}[C_n(t)-\Theta_{nk}^{2n}C_k(t)]\\&-\sum_{n=1}^{\infty}\frac{\langle\hat{N}_2^n\rangle_0}{2}\sum_{k=0}^{n-1}b_{n,k}[C_n(t)-\Theta_{nk}^{2n-2}C_k(t)]\\&+\sum_{n=1}^{\infty}\frac{\langle\hat{F}^n\rangle_0}{\Omega_n}\sum_{k=0}^{n-1}b_{n,k}[S_n(t)-\Theta_{nk}^{2n-1}S_k(t)]\,,\end{aligned} \qquad (41)$$

where

$$\langle\hat{\Lambda}^k\rangle_0=\tfrac{1}{2}\epsilon^2\langle\hat{N}_1^{k+1}\rangle_0+\alpha\langle\hat{I}^k\rangle_0+\tfrac{1}{2}\epsilon^2(k+1)\langle\hat{N}_{2,1}^k\rangle_0\,,$$

$$b_{n,k}\equiv a_{n,k}\Theta_{kn}^{2n-2}\,,$$

$$C_j(t)\equiv\cos(\Omega_j t)-1\,,\quad S_j(t)\equiv\sin(\Omega_j t)\,,$$

$$a_{n,k}=\frac{(-1)^{n+k+1}}{(n-k)!k!}\,.$$

A remarkable property of eq. (41) is that the first nonvanishing term of the correlation $\langle\hat{O}_i^n\rangle_0$ in this solution is proportional to t^{2n}, t^{2n+1}, or t^{2n+2}, depending on the different operators $\hat{O}_i$. This can be seen if one makes a Taylor expansion of $\langle\hat{O}_i^n\rangle_t$ uses eqs. (22)–(27) and the fact that the $a_{n,k}$ satisfies the following set of linear equations,

$$\sum_{k=0}^{n-1}a_{n,k}(n-k)k^i=\delta_{i,n-1}\,,\quad i=0,\dots,n-1\,. \qquad (42)$$

Thus, up to a given time, there will be only a finite number of correlations that will contribute substantially to the solution (41) since the others will be negligible. Similar expressions can be obtained for all the operators.

In order to show how the MEP formalism works, we will also evaluate the initial conditions for a particular case. Let us consider the special situation, broadly used in the literature, of having one particle in level 1, zero particles in level 2, an electromagnetic field with mean number of photons $\langle\hat{\Delta}^0\rangle_0\neq0$ (i.e. $\lambda_6^0(0)\neq0$), and $\alpha=0$ (i.e. resonance). Notice that in resonance

$$\Theta_{nk}=\sqrt{(n+1)/(k+1)}\,.$$

From eqs. (8), (31) we obtain that

$$\langle\hat{N}_2^n\rangle_0=0\,,\quad\langle\hat{F}^n\rangle_0=0\,,\quad\langle\hat{I}^n\rangle_0=0\,,$$

$$\langle\hat{N}_{2,1}^n\rangle_0=0\,,\quad\langle\hat{N}_1^0\rangle_0=1\,,$$

$$\langle\hat{N}_1^n\rangle_0=\langle\hat{\Delta}^{n-1}\rangle_0=\frac{n!}{\{\exp[\lambda_6^0(0)]-1\}^n}\,.$$

Thus, using eq. (41) we obtain

$$\begin{aligned}\langle\hat{N}_1^0\rangle_t=1+\frac{1}{2}\bigg\{&\frac{C_0(t)}{\exp[\lambda_6^0(0)]-1}\\&+\sum_{n=1}^{\infty}\frac{n!}{\{\exp[\lambda_6^0(0)]-1\}^{n+1}}\sum_{k=0}^{n-1}a_{n,k}\left(\frac{k+1}{n+1}\right)^{n-1}\\&\times\left[C_n(t)-\left(\frac{n+1}{k+1}\right)^n C_k(t)\right]\bigg\}\,.\end{aligned} \qquad (43)$$

After some algebra eq. (43) can be rewritten as

$$\langle\hat{N}_1^0\rangle_t=\frac{1}{2}\left(1+\sum_{n=0}^{\infty}\frac{\langle\hat{\Delta}\rangle_0^n}{(1+\langle\hat{\Delta}\rangle_0)^{n+1}}\cos(2\sqrt{n}|\gamma|t)\right) \qquad (44)$$

Equation (44) is the population number of level 1 when the initial state of the field is a thermal or chaotic one. Observe that λ_6^0 can be identified with $\beta^*\omega$, where β^* is proportional to the inverse of the temperature associated only to the field [14]. Note that $\beta^*\omega$ is a positive number and λ_6^0 must be positive in order to have a normalizable density function. So we can see that the evolution equation of $\langle\hat{N}_1^0\rangle_t$, even when the example considered is extremely simple, depends on the correlations of all orders between $\hat{N}_1^0$ and the field. These correlations are non-zero and cannot be arbitrarily chosen notwithstanding that the two levels and the field are initially decoupled.

Summarizing, we have presented a generalized version of the JCH giving a description in terms of physically relevant operators. Since an arbitrary function of time has been included, this formalism allows us to study the system even when the coupling is time-dependent. Particularly, the solution presented in ref. [9] is also included in our formalism, although we have left for an extended paper a detailed comparison. The advantages of our approach results from the following facts: (a) we have given a description of the system in terms of three sets of physically relevant operators giving a way out to the problem of data given in terms of different physical magnitudes; (b) for one of these sets of relevant operators the temporal evolution equations have been shown; (c) we have obtained new invariants of motion in terms of the Lagrange multiplier (i.e. intensive variables) which can only be constructed using MEP; (d) the invariants we have found restrict the possible values of the initial conditions: (e) these initial mean values have been properly evaluated using a MEP density operator (as was pointed out previously [1,11], the initial conditions play a role as important as the dynamics itself, although not all the formalisms are in the position to distinguish clearly which are the pertinent and coherent set of initial conditions); (f) an extension of a sort of quantized version of the Bloch sphere in the dual space of Lagrange multipliers has been obtained, converting the original non-commutative operator's structure into geometrical relationships (see eqs. (36)–(38)); (g) the importance of the correlations in the *generalized time-independent* JCH has been shown. Finally, we want to stress the fact that using the MEP approach we have naturally obtained that the *n*th-order coherence functions of the field (see for example ref. [13], pp. 327–330) are relevant operators.

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