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CONDUCTIVITY OF SUPERCONDUCTING CHALCOGENIDES

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A theoretical study of a superconducting transition in many valley semiconductors, allowing interband electron-electron pairing, is presented. The order parameter, at very low temperatures, and the ac-conductivity are calculated and shown to reproduce known results in the limit when the "dielectric gap" is going to zero.

The problem of superconducting transitions in many-valley semiconductors and semimetals was first studied by Gurevich et al[1] and by Cohen[2]. The transition was expected since intervalley phonon processes (large momentum transfer) would give a large attractive electron-electron interaction, because the screening in this case is less than in intravalley processes. Furthermore, the existance of many equivalent valleys in a degenerate semiconductor provides a large number of states into which an electron could scatter through an intervalley phonon processes. Based on those assumptions superconducting transitions were sought in several materials and found in GeTe, SnTe, $SrTiO_3$, $Ba_xSr_{1-x}TiO_3$, $Ca_xSr_{1-x}TiO_3$ and $T\ell BiTe_2$ with very low transition temperatures and small superconducting gaps[3]. However, attempts to observe bulk superconductivity in the lead chalcogenides were unsuccessful. Recently, the subject of superconductivity in many valley semiconductors gained new life with the report of Chernik and Lykov[4] who found that PbTe doped with Tℓ has a superconducting transition at temperatures as high as 1.4 K. On the theoretical side, however, very little was achieved aside from the static properties of the system.[1,2,5,6]

In this work we present a calculation of the a.c.-conductivity of superconducting semiconductors, based on a BCS type interaction of electrons in different valleys. We have neglected intravalley electron pairing since screening would be strong. The Hamiltonian of the system for two equivalent valleys can be written as:

$$H = \sum_{\alpha=1,2} \int \varepsilon_\alpha (\vec{p}) \Psi^+_{\alpha\sigma}(\vec{r}) d\vec{r} +$$

$$+ \sum_{\sigma,\sigma'} \lambda \int \Psi^+_{1\sigma}(\vec{r}) \Psi^+_{2\sigma'}(\vec{r}\,') \Psi_{2\sigma'}(\vec{r}\,') \Psi_{1\sigma}(\vec{r}) d\vec{r} \ , \quad (1)$$

where the indices α, σ and σ' are the band and spin indices respectively, $\varepsilon_\alpha(\vec{p})$ is the single particle energy in band α, measured from the Fermi surface, with minima located at $\pm k_L$. λ is the interaction strength between electrons of equivalent bands, and $\Psi_{\alpha\sigma}(\vec{r})$ is the electron field operator (We are using $\hbar = c = k_B = 1$). The extension to n equivalent valleys is straightforward and can be carried as proposed in Ref. 2. We define a set of Green's functions for such a system as:

$$G_{\alpha\beta}(\vec{x},\vec{x}\,') = -i\langle \tau(\Psi_{\alpha\sigma}(\vec{x})\Psi_{\beta\sigma}(\vec{x}\,'))\rangle \ , \quad (2)$$

$$F^+_{\alpha\beta}(\vec{x},\vec{x}\,') = -i\langle \tau(\Psi^+_{\alpha,-\sigma}(\vec{x})\Psi^+_{\beta,\sigma}(\vec{x}\,'))\rangle \ ,$$

where τ is the time ordering operator. Considering the two equivalent valleys, as minima of only one band separated by a very high barrier, the electrons which form pairs are those near the Fermi surface with momentum $\vec{p}$ and $-\vec{p}$ respectively. Under this assumption:

$$\varepsilon_1(\vec{p}) = \frac{(\vec{p}-\vec{k}_L)^2 - p_F^2}{2m^*} = \varepsilon_2(-\vec{p}) = \frac{(-\vec{p}+\vec{k}_L)^2 - p_F^2}{2m^*} \ , \quad (3)$$

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where m^* is the electronic effective mass. Calling $\varepsilon = \varepsilon_1 + \lambda n$, with

$$n_\alpha(\vec{x}) = \sum_\sigma \langle \Psi^+_{\alpha\sigma}(\vec{x})\Psi_{\alpha\sigma}(\vec{x})\rangle \quad ,$$

we find:

$$G_{11}(\vec{p},\omega) = \left[(\omega^2-\varepsilon^2)(\omega+\varepsilon)+\Delta_d^2(\omega-\varepsilon)-\Delta_2^2(\omega+\varepsilon)\right].D^{-1} \quad , \tag{4}$$

$$G_{21}(\vec{p},\omega) = i\left[\Delta_d^*(\omega+\varepsilon)^2+\Delta_d^3+\Delta_d^*\Delta_2^2\right].D^{-1} \quad , \tag{5}$$

$$F^+_{21}(\vec{p},\omega) = -\Delta_2^*\left[\omega^2+\varepsilon^2+\Delta_d^2+\Delta_2^2\right].D^{-1} \quad , \tag{6}$$

$$F^+_{11}(\vec{p},\omega) = -\Delta^*_d\left[\Delta_2(\omega+\varepsilon)-\Delta_2^*(\omega-\varepsilon)\right].D^{-1} \quad , \tag{7}$$

$$D = (\omega^2-\varepsilon^2)^2+2\Delta_d^2(\omega^2+\varepsilon^2)-2\Delta_2^2(\omega^2-\varepsilon^2)+(\Delta_d^2+\Delta_2^2)^2 \quad , \tag{8}$$

with:

$$\Delta_d = \lambda\; G_{21}(0) \tag{9}$$

$$\Delta_2^* = \lambda\; F^+_{21}(0) \; . \tag{10}$$

Solving this set of functions self-consistently we find for the quasi-particle spectrum:

$$\omega = \pm\left(\sqrt{\varepsilon^2+\Delta^2}\right) \pm i\,\Delta_d) \quad . \tag{11}$$

At zero temperature, the order parameter satisfies:

$$1 = \frac{\lambda}{(2\pi)^2\,|v_F-\lambda\bar{\nabla}n|}\;\ell n\left|\frac{4(\omega_c-i\Delta_d)^2-\Delta_2^2}{\Delta_2^2}\right| \tag{12}$$

with ω_c, a cut-off frequency. For finite temperature, the expression becomes:

$$1 = \frac{-\lambda}{(2\pi)^2|v_F-\lambda\bar{\nabla}n|}\int_0^{\omega_c}\frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta_2^2}}\left\{\mathrm{tgh}\left(\frac{\sqrt{\varepsilon^2+\Delta_2^2}+i\Delta_d}{2T}\right) + \mathrm{tgh}\left(\frac{\sqrt{\varepsilon^2+\Delta_2^2}-i\Delta_d}{2T}\right)\right\} \quad .$$

Both results reduce to the BCS limit, for $\Delta_d \to 0$.[7] Δ_d, known as the "dielectric gap", can be interpreted as an inverse lifetime of the quasi-particle produced by the competition between forming an electron-electron pair, or an electron-hole pair.

Next we calculate the response function to an ac-field. Following the procedure, as well as the notation of Ref. 7, we have found the response function to be:

$$Q(\vec{k},\omega) = \frac{3T\pi^2}{4v_F|k|}\sum_{\omega_n}\left\{1+\frac{1}{4}\left[\frac{\Delta_2^2(\beta^++\beta^-)(\gamma^++\gamma^-)}{\beta^+\beta^-\gamma^+\gamma^-} - \left(\frac{\omega_--\Delta_d}{\gamma^-}+\frac{\omega_-+\Delta_d}{\gamma^+}\right)\left(\frac{\omega_+-\Delta_d}{\beta^-}+\frac{\omega_++\Delta_d}{\beta^+}\right)\right]\right\} \tag{13}$$

here $\omega_n = (2n+1)\pi T$, and

$$\beta^\pm = \sqrt{(\omega_+ \pm \Delta_d)^2 + \Delta_2^2} \tag{14}$$

$$\gamma^\pm = \sqrt{(\omega_- \pm \Delta_d)^2 + \Delta_2^2} \quad . \tag{15}$$

In conclusion, we have found an analytic expression for the response function, which reproduces well known results[7] when the dielectric gap goes to zero. Eq. (13) can be used to obtain the resistivity to compare with the measurements in GeTe[3] and on PbTe:Tℓ[4]. Extensions of this work are in progress.

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